Fundamental Strings are Massless Waves

PhD Student Triangle

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Outline

Motivation

Kaluza-Klein Theory

- Start with massless, uncharged state in full theory
- States in reduced theory have mass and charge
- Given by momentum in KK direction

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Example

- Null wave solution in M-theory gives D0-brane
- D0-brane is momentum mode in 11th direction
- Mass and charge given by momentum - BPS state

Standard Solutions

- pp-wave

D0-Brane

$$\mathrm{d}s^2 = -H^{-1}\mathrm{d}t^2 + H\left[\mathrm{d}z - (H^{-1} - 1)\mathrm{d}t\right]^2 + \mathrm{d}\vec{y}^2_{(D-2)}$$

$$B_{\mu\nu} = 0, \qquad e^{-2\phi} = e^{-2\phi_0}$$

- F1-string

$$\mathrm{d}s^2 = -H^{-1}\left[\mathrm{d}t^2 - \mathrm{d}z^2\right] + \mathrm{d}\vec{y}^2_{(D-2)}$$

$$B_{tz} = -(H^{-1} - 1), \qquad e^{-2\phi} = He^{-2\phi_0}$$

- Harmonic Function

$$H = 1 + \frac{h}{|\vec{y}_{(D-2)}|^{D-4}}, \qquad \nabla^2 H = 0$$

Introduction to Double Field Theory

Novel formulation of string theory

- Bosonic NS-NS sector: $g_{\mu\nu}$, $B_{\mu\nu}$ and ϕ
- Makes T-duality a manifest symmetry of the action
- Metric and B-field on equal footing - geometric unification

T-Duality

Winding modes

- Strings can wind around compact dimensions
- Winding modes $\leftrightarrow$ momentum modes in dual space
- Mass spectrum of closed string in circle with radius R

$$M^2 = (N + \tilde{N} - 2) + p^2 \frac{\ell_s^2}{R^2} + \tilde{p}^2 \frac{\ell_s^2}{\tilde{R}^2}$$

- Invariant under $R \leftrightarrow \tilde{R} = \frac{\ell_s^2}{R}$ and $p \leftrightarrow \tilde{p}$

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Combine space and dual space

- $O(D,D)$ structure group $\rightarrow$ invariant metric $\eta = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Geometric Framework

Doubling the dimension of space to $2D$

- Include winding coordinates $\tilde{x}_\mu$
- Need section condition to pick D dimensions

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Unification of two concepts

- Metric and B-field $\rightarrow$ generalized metric
- Diffeos and gauge transformations $\rightarrow$ generalized diffeos
- Generated by generalized Lie derivative

The Doubled Formalism

Generalized coordinates

- Combine x^μ and $\tilde{x}_\mu$ into

$$X^M = (x^\mu, \tilde{x}_\mu)$$

- $\mu = 1, \dots, D$ and $M = 1, \dots, 2D$

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- $\mu = 1, \ldots, D$ and $M = 1, \ldots, 2D$

Generalized metric

- Combine metric $g_{\mu\nu}$ and Kalb-Ramond field $B_{\mu\nu}$ into

$$\mathcal{H}_{MN} = \begin{pmatrix} g_{\mu\nu} - B_{\mu\rho} g^{\rho\sigma} B_{\sigma\nu} & B_{\mu\rho} g^{\rho\nu} \\ -g^{\mu\sigma} B_{\sigma\nu} & g^{\mu\nu} \end{pmatrix}$$

- Rescale the dilaton $\qquad e^{-2d} = \sqrt{g} e^{-2\phi}$

The DFT Action

The action integral

$$S = \int \mathrm{d}^{2D} X e^{-2d} R$$

The Ricci scalar

$$\begin{aligned} R = &\frac{1}{8}\mathcal{H}^{MN}\partial_M\mathcal{H}^{KL}\partial_N\mathcal{H}_{KL} - \frac{1}{2}\mathcal{H}^{MN}\partial_M\mathcal{H}^{KL}\partial_K\mathcal{H}_{NL} \\ &+ 4\mathcal{H}^{MN}\partial_M\partial_N d - \partial_M\partial_N\mathcal{H}^{MN} \\ &- 4\mathcal{H}^{MN}\partial_M d\partial_N d + 4\partial_M\mathcal{H}^{MN}\partial_N d \end{aligned}$$

Equations of Motion

Since $\mathcal{H}$ is constrained, get projected EoMs

$$P_{MN}{}^{KL} K_{KL} = 0$$

where

$$K_{MN} = \delta R / \delta \mathcal{H}^{MN}$$

$$P_{MN}{}^{KL} = \frac{1}{2}(\delta_M{}^{(K} \delta_N{}^{L)} - \mathcal{H}_{MP} \eta^{P(K} \eta_{NQ} \mathcal{H}^{L)Q})$$

Dilaton equation

$$R = 0$$

The DFT Wave Solution

$$X^M = (t, z, y^m, \tilde{t}, \tilde{z}, \tilde{y}_m)$$

Generalized metric

$$\begin{aligned} \mathrm{d}s^2 &= \mathcal{H}_{MN}\mathrm{d}X^M\mathrm{d}X^N \\ &= (H-2)\left[\mathrm{d}t^2 - \mathrm{d}z^2\right] - H\left[\mathrm{d}\tilde{t}^2 - \mathrm{d}\tilde{z}^2\right] \\ &\quad + 2(H-1)\left[\mathrm{d}t\mathrm{d}\tilde{z} + \mathrm{d}\tilde{t}\mathrm{d}z\right] \\ &\quad + \delta_{mn}\mathrm{d}y^m\mathrm{d}y^n + \delta^{mn}\mathrm{d}\tilde{y}_m\mathrm{d}\tilde{y}_n \end{aligned}$$

Rescaled dilaton

$$d = const.$$

The DFT Wave Solution

Properties

- No mass, null-like
- Carries momentum in $\tilde{z}$ direction
- Interprete as null wave in DFT
- Smeared over dual directions $\rightarrow$ obeys section condition

Reducing the Solution

KK-Ansatz to reduce dual directions

- Get fundamental string solution
- Extended along z
- Mass and charge given by momentum in $\tilde{z}$

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KK-Ansatz to reduce dual directions

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If z and $\tilde{z}$ are exchanged

- Get pp-wave in z direction
- Expected as wave and string are T-dual

Key Result

The fundamental string is a massless wave in doubled space with momentum in a dual direction.

Goldstone Mode Analysis

Zero modes

- Symmetry breaking
- Moduli $\rightarrow$ collective coordinates
- Generated by large gauge transformations / diffeos
- Make local on worldvolume $\rightarrow$ get zero modes

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Number of modes

- String: $D-2$ modes
- Doubled wave / string: ???

Constructing the Zero Modes

Transformations of $\mathcal{H}$ and d

$$h_{MN} = \mathcal{L}_\xi \mathcal{H}_{MN} \qquad \lambda = \mathcal{L}_\xi d$$

- gauge parameter $\xi^M = (0, H^\alpha \hat{\phi}^m, 0, H^\beta \hat{\tilde{\phi}}_m)$
- $\hat{\phi}^m$ and $\hat{\tilde{\phi}}_m$ are constant moduli

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Allow dependece on $x^a = (t, z)$ to get zero modes

$$\hat{\phi}^m \to \phi^m(x) \qquad \hat{\tilde{\phi}}_m \to \tilde{\phi}^m(x)$$

Equations of motion

- Insert into DFT EoMs (two derivatives, first order)
- Find $\Box\phi = 0$ and $\Box\tilde{\phi} = 0$
- Also get self-duality relation for $\Phi^M = (0, \phi^m, 0, \tilde{\phi}_m)$

$$\mathcal{H}_{MN} \mathrm{d}\Phi^N = \eta_{MN} \star \mathrm{d}\Phi^N$$

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Duality symmetric string in doubled space (Tseytlin)

- Can be written as (anti-)chiral equation for $\psi_\pm = \phi \pm \tilde{\phi}$

$$\mathrm{d}\psi_\pm = \pm \star \mathrm{d}\psi_\pm$$

Summary

Wave solution in DFT

- Solution unifies pp-wave and F1-string (T-duals)
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Goldstone modes

- Find chiral zero modes on wave solution
- Correct degrees of freedom for doubled string

Other DFT Solutions

T-dual objects in string theory

- F1-string and pp-wave - fundamental
- NS5-brane and KK-monopole - solitonic
- D-branes? (Problem in DFT: couple to R-R forms)

NS5-brane / KK-monopole

▸ NS5, KK

- KK-circle in a dual direction, say $\tilde{z}$
- Periodic array of NS5-branes $\rightarrow$ smeared along z

Extension to M-Theory

Extended theories

- Make U-duality manifest
- Include wrapping directions
- Geometrically unify metric and C-field(s)

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Example: $SL(5)$

- Duality group for M-theory in 4 dimensions x^{μ}
- Combine with 6 wrapping directions $y_{\mu\nu}$
- Wave in extended space gives M2-brane

Other Solutions

- D0-brane

Back to Standard Solutions

$$ds^2 = -H^{-1}dt^2 + d\vec{y}^2_{(d-1)}, \qquad A_t = -(H^{-1} - 1)$$

- KK-monopole

Back to Other Solutions

$$ds^2 = -dt^2 + d\vec{x}^2_{(d-5)} + H^{-1}\left[dz + A_i dy^i\right]^2 + H d\vec{y}^2_{(3)}$$

$$\partial_{[i}A_{j]} = \frac{1}{2}\epsilon_{ij}{}^k\partial_k H, \qquad e^{-2\phi} = e^{-2\phi}$$

- NS5-brane

$$ds^2 = -dt^2 + d\vec{x}^2_{(d-5)} + H d\vec{y}^2_{(4)}, \quad B_{zi} = A_i, \quad e^{-2\phi} = H^{-1}e^{-2\phi_0}$$