MSci PHY7005/QMUL PHY-415 Relativistic Waves and Quantum Fields

Problem Set 1

(1) Consider the following action for the complex field $\psi(t, \vec{x})$

$$S = \int dt d^3\vec{x} \left[i\psi^* \frac{\partial \psi}{\partial t} - \frac{1}{2\mu} \sum_{j=1}^{3} \frac{\partial \psi^*}{\partial x^j} \frac{\partial \psi}{\partial x^j} - V(t, \vec{x}) |\psi|^2 \right] ,$$

where $V(t, \vec{x})$ is a function and μ a constant.

Treat ψ and ψ^* as independent variables and derive the equation of motions. **[3 marks]**

Show that the transformation $\psi(t, \vec{x}) \to e^{i\theta}\psi(t, \vec{x})$ is a global symmetry (i.e. it leaves S invariant only when θ is constant). **[1 marks]**

Apply Noether's theorem and derive the associated conserved charge **[4 marks]**

(*) Check explicitly that it is conserved **[2 marks]**

(2) σ_1 is the Pauli matrix $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. Explain why it can represent a Quantum Mechanical operator acting on (the Hilbert space) C^2? **[1 marks]**

Derive the explicit form of the 2×2 matrix $O = e^{i\theta\sigma_1}$. **[4 marks]**

What type of matrix is O? **[2 marks]**

(3) For every three operators or matrices A, B, C show that

$$[A, BC] = [A, B]C + B[A, C] \quad \text{and}$$

$$[[A, B], C] = [A, [B, C]] + [[A, C], B] = \{A, \{B, C\}\} - \{\{A, C\}, B\}$$

where $\{A, B\} \equiv AB + BA$ is the anti-commutator. **[3 marks]**