MSci PHY7005/QMUL PHY-415 Relativistic Waves and Quantum Fields

Problem Set 10

From the 2012 Exam paper.

1. Consider the theory of one real ϕ_r and one complex scalar field ϕ_c defined by the action

$$S = \int \left\{ \frac{1}{2}\partial_\mu \phi_r \partial^\mu \phi_r + \partial_\mu \phi_c^\dagger \partial^\mu \phi_c - \frac{1}{2} m^2 \phi_r^2 - \lambda \phi_c^\dagger \phi_c \phi_r \right\} d^4x \ .$$

Define the free Lagrangian density $\mathcal{L}_0$ and the interaction part $\mathcal{L}_{\text{int}}$. Give the physical units for the ϕ's and λ so as to make the action dimensionless (use the conventions $c = \hbar = 1$).

[4 marks]

2. Write the Dyson formula for the S-matrix in terms of $\mathcal{L}_{\text{int}}$.

[4 marks]

3. Use the Fourier expansions

$$\phi_r(x^\mu) = \int \frac{d^3k}{\sqrt{2E_{\vec{k}}}(2\pi)^3} \left[a(\vec{k})\, e^{-ik\cdot x} + a^\dagger(\vec{k})\, e^{ik\cdot x} \right] ,$$

$$\phi_c(x^\mu) = \int \frac{d^3k}{\sqrt{2|k|}(2\pi)^3} \left[b(\vec{k})\, e^{-ik\cdot x} + c^\dagger(\vec{k})\, e^{ik\cdot x} \right] .$$

The field ϕ_c creates a type of particles (c-particles) and destroys another type of particles (b-particles); the field ϕ_r creates and destroys a-particles. Give at least one reason why b or c particles cannot decay into a-particles.

[4 marks]

4. Consider the decay of an a-particle of momentum $\vec{p}_3$ into a b-particle and c-particle of momenta $\vec{p}_1$ and $\vec{p}_2$ respectively.

   - Write down the in- and out-states using the Fourier modes and the vacuum state $|0\rangle$.

   [3 marks]

   - Focus on the first term in Dyson's formula that contributes to the decay process mentioned above. Write this term as a function of the Fourier modes and calculate the decay amplitude.

   [5 marks]