MSci PHY7005/QMUL PHY-415 Relativistic Waves and Quantum Fields

Problem Set 3

1. Consider a Klein-Gordon field of the form

$$\phi(t, x^i) = N\mathrm{e}^{-\mathrm{i}\frac{mc^2}{\hbar}t}\,\psi(t, x^i)\ ,$$

where N is a constant and m is the mass. Start from the Klein-Gordon equation for $\phi(t, x^i)$ and derive an equation for $\psi(t, x^i)$. **[4 marks]**

Consider the case where $\psi(t, x^i)$ is slowly varying in time and you can neglect $\hbar\partial_t\psi$ with respect to $mc^2\psi$. Show that the equation for ψ reduces to the usual Schrödinger equation for a free particle. **[1 marks]**

Take ψ to be a constant, then

$$\phi_m(t, x^i) = N\mathrm{e}^{-\mathrm{i}\frac{mc^2}{\hbar}t}\ .$$

Show that $\phi_m(t, x^i)$ above is a solution of the Klein-Gordon equation. **[1 marks]**

Perform a boost and derive the $\phi'_m(t', x'^i)$ in the new coordinate frame. **[3 marks]**

Show that ϕ'_m solves the Klein-Gordon equation. **[3 marks]**

2. Which components of the (three-dimensional) momentum operator $\vec{P}$ and angular momentum operator $\vec{L}$ correspond to conserved quantities (constants of motion) for the following QM Hamiltonian? (Set $\hbar = 1$).

$$H = \frac{\vec{P}^2}{2m} + \frac{1}{2}m\omega^2(x^2 + y^2)\ .$$

[5 marks]

Consider the wavefunction $\psi(t, x^i) = \mathrm{e}^{-iEt+ipz}e^{-\frac{m\omega}{2}(x^2+y^2)}$ and find the relation between E and p which ensures that ψ solves Schroedinger's equation. **[1 marks]**

Perform a rotation of an angle θ around the x-axis and derive the corresponding ψ'. Is ψ' a solution of the original Schroedinger's equation? **[2 marks]**