MSci PHY7005/QMUL PHY-415 Relativistic Waves and Quantum Fields

Problem Set 4

1. Consider the theory of one real scalar field ϕ_r of mass m and one complex scalar field ϕ_c defined by the action

$$S = \int \left\{ \frac{1}{2}\partial_\mu \phi_r \partial^\mu \phi_r + \partial_\mu \bar{\phi}_c \partial^\mu \phi_c - \frac{1}{2} m^2 \phi_r^2 - \lambda \bar{\phi}_c \phi_c \phi_r \right\} d^4x \ ,$$

where λ is a constant and we used the conventions $c = \hbar = 1$. Define the free Lagrangian density $\mathcal{L}_0$ and the interaction part $\mathcal{L}_{\text{int}}$. Give the physical units for the ϕ's and λ so as to make the action dimensionless. **[5 marks]** (From the 2012 exam paper)

Use the Euler-Lagrange equations and derive the equation of motions. **[5 marks]**

2. Start from Dirac's equation in four space-time dimensions

$$i\hbar \frac{\partial \psi}{\partial t} = -i\hbar c \alpha^i \frac{\partial \psi}{\partial x^i} + mc^2 \beta \psi$$

and motivate the relations

$$\left\{\alpha^i, \alpha^j\right\} = 2\delta^{ij}\mathbb{I}_4 \ , \quad \left\{\alpha^i, \beta\right\} = 0 \ , \quad \beta^2 = \mathbb{I}_4 \ ,$$

where $\mathbb{I}_4$ is the 4×4 identity matrix. **[5 marks]** (From the 2012 exam paper)

Show that a new set of matrices related to α^i and β by a unitary transformation U

$$\tilde{\alpha}^i = U \alpha^i U^\dagger \ , \quad \tilde{\beta}^i = U \beta^i U^\dagger$$

satisfies the same anticommutation relations above. **[2 marks]**

Write explicitly a set of 4×4 matrices satisfying the anticommutation relations above. **[3 marks]**