MSci PHY7005/QMUL PHY-415 Relativistic Waves and Quantum Fields

Problem Set 5

In the following set $\hbar = c = 1$.

1. Use Dirac's representaton of the Gamma matrices and show that, if Ψ obeys Dirac's equation in four space-time dimensions
$$(i\gamma^\mu \partial_\mu - m)\Psi = 0 \ ,$$
then $\overline{\Psi} = \Psi^\dagger \gamma^0$ obeys the equation $i\partial_\mu \overline{\Psi}\gamma^\mu + m\overline{\Psi} = 0$. **[4 marks]**

2. Consider the Dirac field $\Psi(x)$ in four space-time dimension. Under parity
$$x^0 \to x'^0 = x^0 \ , \qquad x^j \to x'^j = -x^j \ ,$$
$\Psi(x)$ transforms as $\Psi'(x') = P\Psi(x)$ with $P = \gamma^0$. Derive the transformation under parity of the bilinear $\bar{\Psi}\gamma^5\Psi$, where $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$. **[3 marks]**

   Show that under, a Lorentz transformation Λ, the following identity
$$\Lambda_s^{-1}\gamma^5\Lambda_s = \gamma^5 \,\mathrm{det}\Lambda$$
is true, where Λ_s is the spinor representation of Λ. **[4 marks]**

3. The Lorentz generators in the spinor representation are
$$\frac{i}{4}[\gamma^\mu, \gamma^\nu]$$
Find the matrix representation a rotation of 2π in the 1-2 plane. **[4 marks]**