MSci PHY7005/QMUL PHY-415 Relativistic Waves and Quantum Fields

Problem Set 7

In the following set $\hbar = c = 1$.

1. Consider the 4-spinors

$$u_r(\vec{p}) = \sqrt{\frac{E+mc^2}{2mc^2}} \begin{pmatrix} \phi_r \\ \frac{\sigma^j p^j c}{E+mc^2}\phi_r \end{pmatrix} \ , \quad v_r(\vec{p}) = \sqrt{\frac{E+mc^2}{2mc^2}} \begin{pmatrix} \frac{\sigma^j p^j c}{E+mc^2}\chi_r \\ \chi_r \end{pmatrix} \ ,$$

where $\sigma^{j=1,2,3}$ are the Pauli matrices and ϕ_r, χ_r are the 2-spinors

$$\phi_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \ , \quad \phi_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \ , \quad \chi_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \ , \quad \chi_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \ .$$

Use Dirac's representation of the gamma matrices and calculate $u_r^\dagger(\vec{p})u_s(\vec{p})$, $\bar{u}_r(\vec{p})u_s(\vec{p})$, $\bar{v}_r(\vec{p})v_s(\vec{p})$, and $\bar{u}_r(\vec{p})v_s(\vec{p})$. **[5 marks]**

If $u_{r\,a}(\vec{p})$ indicates the a^{th} component of $u_r(\vec{p})$, show that

$$\sum_{r=1}^{2} \left(u_{r\,a}(\vec{p})\bar{u}_{r\,b}(\vec{p}) - v_{r\,a}(\vec{p})\bar{v}_{r\,b}(\vec{p}) \right) = \delta_{ab} \ ,$$

ie the parenthesis is just the identity matrix in the spinor space. **[5 marks]**

2. Start from the free Klein-Gordon action S_{KG} for a complex scalar field and use the minimal coupling presecription to write the action $S^{(A)}_{KG}$ in presence of a classical vector field A_μ. **[3 marks]**

Isolate in $S^{(A)}_{KG}$ the term linear in A_μ and show that it has the form $A_\mu j^\mu$, where j^μ is the Noether current corresponding to the symmetry $\phi \to e^{i\theta}\phi$ of S_{KG}. **[3 marks]**

Suppose that A_μ describe a constant magnetic field B in the plane (x^1, x^2), i.e.

$$A_\mu(x) = B\delta_{\mu 2} x^1 \ .$$

Derive from $S^{(A)}_{KG}$ the equations of motion for ϕ. **[4 marks]**