MSci PHY7005/QMUL PHY-415 Relativistic Waves and Quantum Fields

Problem Set 8

In the following set $\hbar = c = 1$.

1. Consider the following two possible realizations of the two dimensional gamma metrices

$$\gamma^0 = \sigma^3 \ , \quad \gamma^1 = i\sigma^2 \ , \quad \hat{\gamma}^0 = \sigma^1 \ , \quad \hat{\gamma}^1 = i\sigma^2 \ .$$

   Find a unitary matrix U such that $\gamma^\mu = U\hat{\gamma}^\mu U^{-1}$, $\mu = 0, 1$. **[6 marks]**

2. Consider the gauge transformation

$$\psi(x) \to \mathrm{e}^{i\theta(x)}\psi(x) \ , \quad A_\mu(x) \to A_\mu(x) + \partial_\mu\theta(x) \ .$$

   Derive the transformation of $D_\mu\psi \equiv \partial_\mu\psi - iA_\mu\psi$. **[3 marks]**

3. Consider a real scalar field (or neutral Klein Gordon field) with creation operators $a^\dagger(\vec{p})$ and annihilation operators $a(\vec{p})$. Using the commutation relations for creation and annihilation operators and assuming that the vacuum state is normalised to one, i.e. $\langle 0|0\rangle = 1$, calculate the following expressions:

$$\langle 0|a(\vec{p}_2)a^\dagger(\vec{p}_1)|0\rangle$$

$$\langle 0|a(\vec{p}_4)a(\vec{p}_3)a^\dagger(\vec{p}_2)a^\dagger(\vec{p}_1)|0\rangle$$

   [5 marks]

4. Consider a free complex Klein Gordon field ϕ; write the Noether current associated to the symmetry $\phi(x) \to \mathrm{e}^{i\theta}\phi(x)$ in terms of the modes. **[6 marks]**