MSci PHY7005/QMUL PHY-415 Relativistic Waves and Quantum Fields

Problem Set 9

In the following set $\hbar = c = 1$.

1. Consider the momentum $\vec{p}$ and the energy $E_p = +\sqrt{p^2 + m^2}$. After a boost in the third direction both the energy and the third component of the momentum change and are indicated with $\vec{p}'$ and $E_{p'}$. Show that

$$\frac{d^3\vec{p}'}{E_{p'}} = \frac{d^3\vec{p}}{E_p}$$

[5 marks]

2. From the exam 2012. The Lagrangian density for a free, real Klein-Gordon field ϕ is

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi\partial^\mu\phi - m^2\phi^2) \; .$$

Obtain the field equation for ϕ; derive the conjugate variable to the field ϕ and write the canonical commutation relations. **[5 marks]**

Use the Fourier expansion

$$\phi(x^\mu) = \int \frac{d^3\vec{k}}{\sqrt{2E_{\vec{k}}}(2\pi)^3}\left[a(\vec{k})\, e^{-ik\cdot x} + a^\dagger(\vec{k})\, e^{ik\cdot x}\right] \; ,$$

and derive the commutation relation among the Fourier modes. **[5 marks]**

Now set the mass to zero, $m = 0$.

- Show that the commutator of two fields at *generic* space-time points x and y

$$i\Delta(x-y) = [\phi(x), \phi(y)]$$

vanishes outside the light-cone (i.e. when $(x-y)^2 \neq 0$). **[5 marks]**

Optional Derive an explicit expression for $\langle 0|\phi(x)\phi(y)|0\rangle$ and show that it does not vanish outside the light-cone. **[0 marks]**