10

$\mathcal{L}_0$ is quadratic in the fields

$$\mathcal{L}_0 = \frac{1}{2}\partial_\mu \varphi_r \partial^\mu \varphi_r - \frac{1}{2} m^2 \varphi_r^2 + \partial_\mu \varphi_c^\dagger \partial^\mu \varphi_c$$

$\mathcal{L}_{int}$ contains the remaining terms

$$\mathcal{L}_{int} = -\lambda \varphi_c^\dagger \varphi_c \varphi_r$$

S should be dimensionless so both φ_r and φ_c must have the same dimension of $\frac{\partial}{\partial x^\mu}$; thus also λ should have dimension of $Length^{-1}$

5b Dyson's equation for the S-Matrix is

$$S = T\left\{ e^{-i\int_{-\infty}^{+\infty} \mathcal{H}_{int}\, dt} \right\},$$ where T is

the time ordering operation and $\mathcal{H}_I$ is the interaction Hamiltonian density

$$\mathcal{H}_{int} = -\int \mathcal{L}_{int}\, d^3x$$

5c

A first reason: the symmetry $\varphi_c \to e^{i\vartheta}\varphi_c$ of the action implies that the difference between the number of b and c particles is conserved. Thus it is not possible to evolve a state with a single b (or c) particle into a state where this particle is absent.

A second reason: φ_c does not have a mass term and thus describe massless particles. By combining energy and momentum conservation with the mass-shell condition we see that a state with $p_\mu = (E, 0, 0\, E)$ can decay only in massless states. However φ_r describes a massive particle so it cannot appear in a decay of b or c particles

5d

1) The only possible decay in a two particle state is

a- particle → b-particle + c-particle

This can be seen either

1) by the conservation rule mentioned in 5c

2) by the explicit form of $\mathcal{L}_{int}$ which contains $\varphi_c^+ \varphi_c \varphi_r$

Then the in state is

$$a^+(\vec{p}_3 = 0)\,|0\rangle = |in\rangle$$

and the out state is

$$b^+(\vec{p}_1)\, c^+(\vec{p}_2)\,|0\rangle = |out\rangle$$

2)

$$1 + iT\left\{\int \mathcal{L}_{int}\, d^4x\right\} + \dots =$$

$$1 + iT\left\{\int \frac{d^3\vec{k}_1 d^3\vec{k}_2 d^3\vec{k}_3}{(2\pi)^9\sqrt{2^3\,|k_1 k_2|\,E_{k_3}}}\lambda\left(b^\dagger(\vec{k}_1)\,e^{ik_1x} + c(\vec{k}_1)\,e^{-ik_1x}\right)\left(b(\vec{k}_2)\,e^{-ik_2x} + c^\dagger(\vec{k}_2)\,e^{ik_2x}\right)\left(a(\vec{k}_3)\,e^{-ik_3x} + a^\dagger(\vec{k}_3)\,e^{ik_3x}\right)\right\} + \dots$$

$$\sim i\int \frac{d^3\vec{k}_1\, d^3\vec{k}_2\, d^3\vec{k}_3}{(2\pi)^9\sqrt{2^3\,|k_1k_2|\,E_{k_3}}}\lambda\, e^{-i(k_3-k_1-k_2)x}$$

$$a(\vec{k}_3)\; b^\dagger(\vec{k}_1)\; c^\dagger(\vec{k}_2) = T_{a\to bc}$$

The decay amplitude is

$$\langle out|T_{a\to bc}|in\rangle = i\lambda\,(2\pi)^4\,\frac{\delta^4(p_3-p_1-p_2)}{\sqrt{2E_{p_3}2|p_1|2|p_2|}}$$