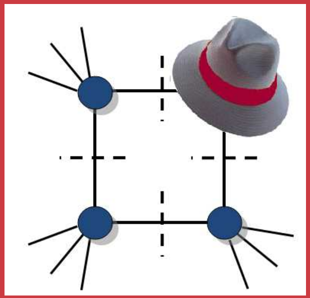
# On-Shell Methods for Collider Physics




**Carola F. Berger**

***CTP, MIT***


**Amplitudes 2010, May 5th, 2010**

# BlackHat and Sherpa



## BlackHat:

**CFB, Zvi Bern, Lance Dixon, Fernando Febres Cordero, Darren Forde, Harald Ita, David Kosower, Daniel Maitre**

BlackHat: arXiv:1004.1659, PRD80 (2009) 074036, PRL102 (2009) 222001, PRD78 (2008) 036003. Badger: JHEP 0901 (2009) 049. Forde: PRD75 (2007) 125019. CFB, Bern, Dixon, Forde, Kosower: PRD74 (2006) 036009.

## Sherpa liaison (real emissions):

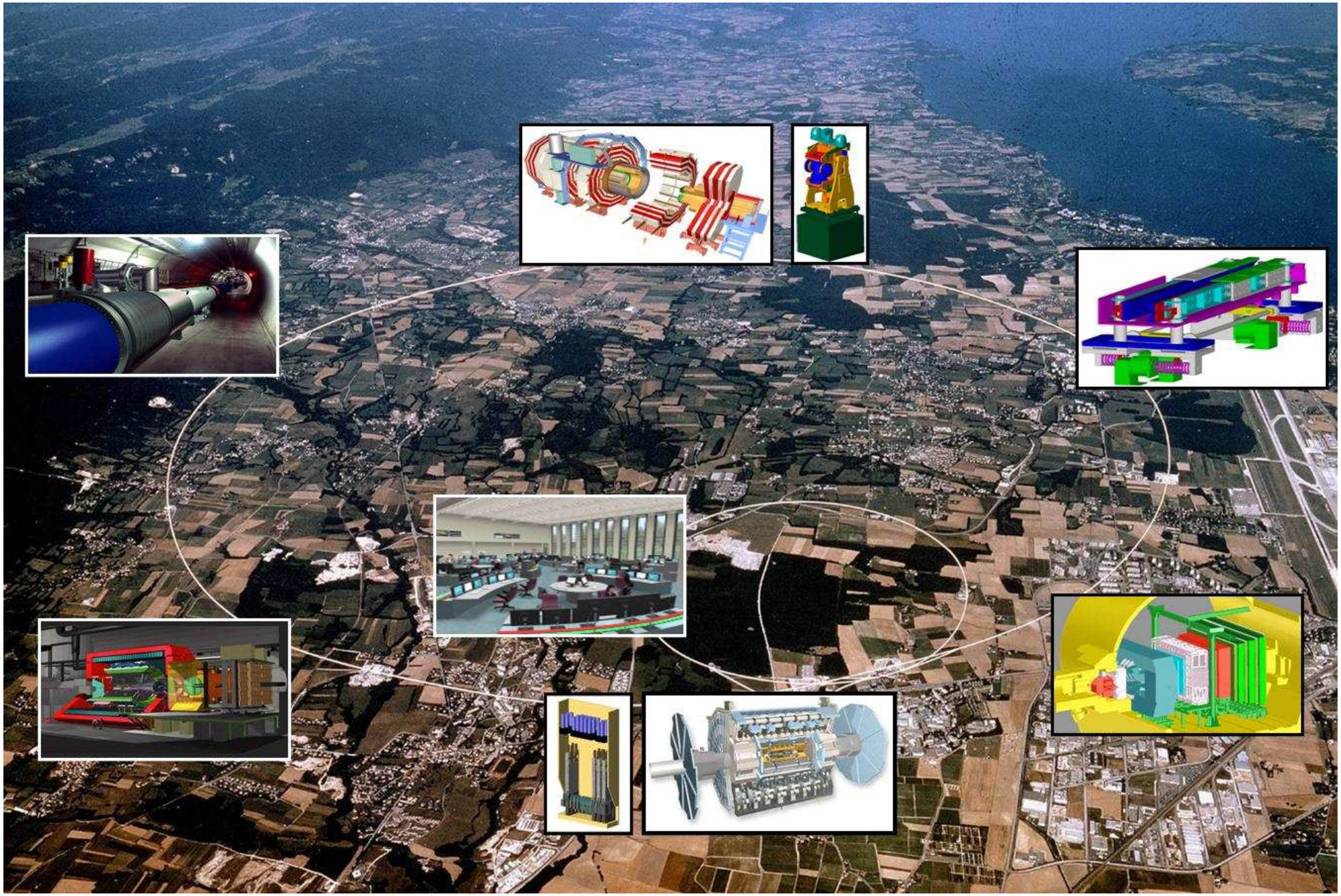
**Tanju Gleisberg**

Gleisberg et al, JHEP 0902 (2009) 007. Gleisberg, Krauss, Eur. Phys. J C53 (2008) 501.

# It's 2010!

- BlackHat and Sherpa
- It's 2010!
- Outline





# It’s 2010!

**Note the black hats**

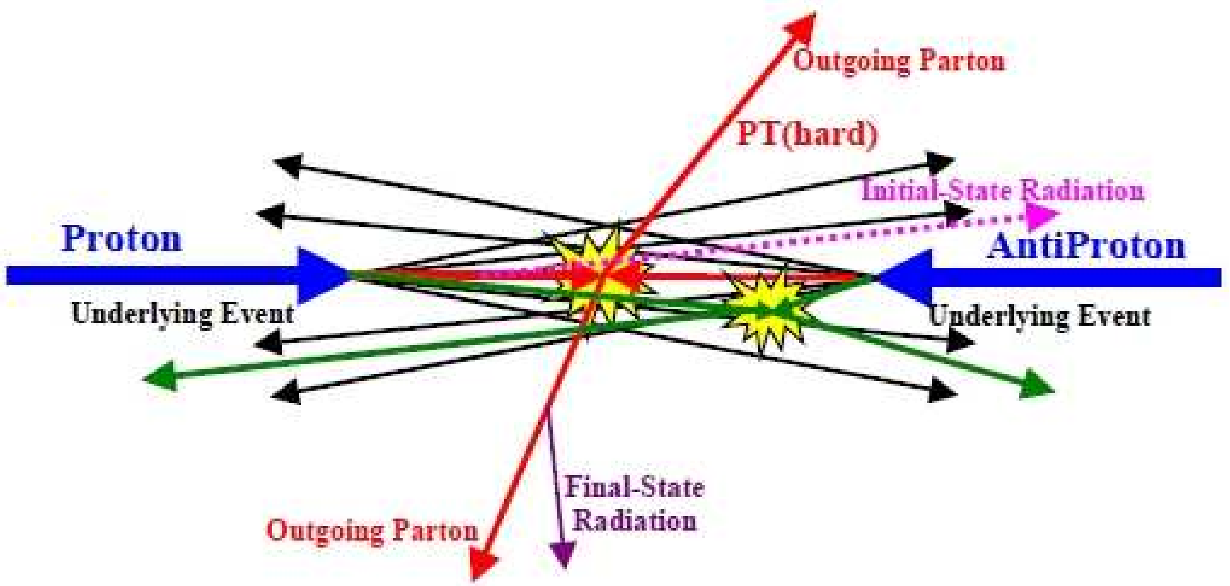
# Outline



- **Introduction - Precision Calculations**
- **On-Shell Methods**
  - **Generalized unitarity**
  - **Rational Terms - recursion and D-dimensional unitarity**
- **Black Magic? BlackHat!**
- **Summary and Outlook**

# Hadron Collisions






$$\sigma = f(x_1) \otimes f(x_2) \otimes \hat{\sigma}(x_1, x_2) \otimes \text{other stuff}$$

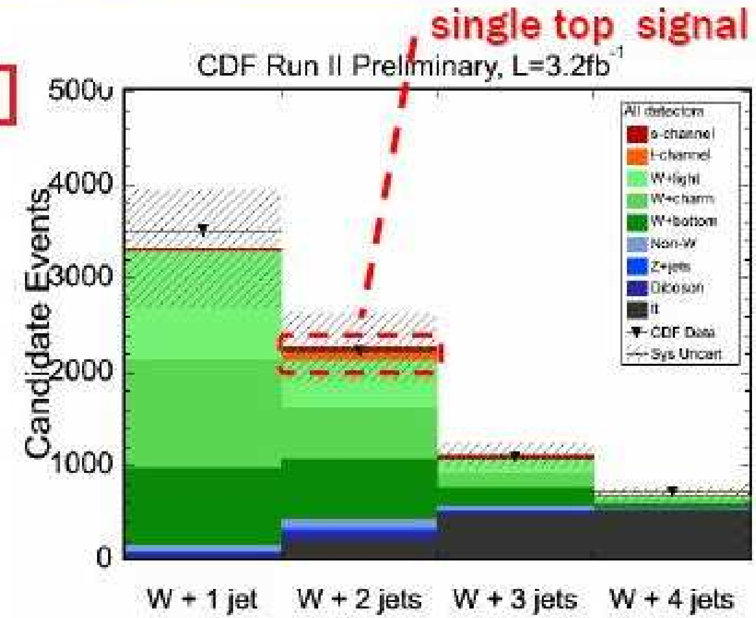
**This talk is about the computation of the hard scattering $\hat{\sigma}$, which is perturbatively calculable for infrared-safe observables**

# Why NLO? Example: Single Top

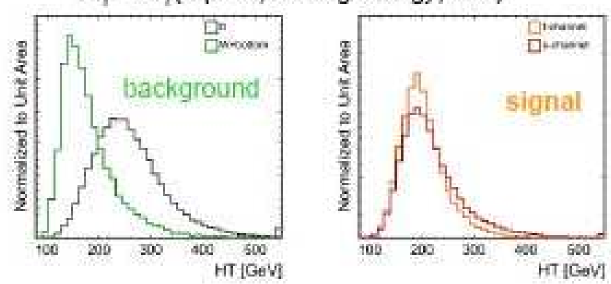


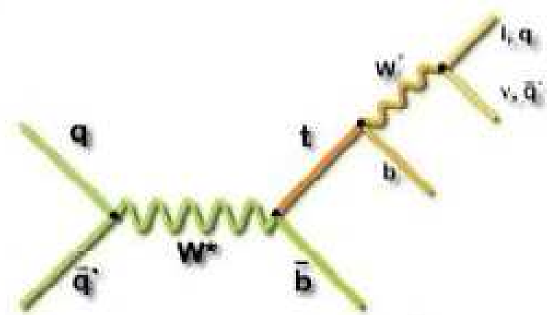
## The Challenge

- "W+2 jet" topology not very distinct
  - Signal/Background W+2jet+btag ~ 1:17
  - Counting experiment impossible
- Many sources of background
  - Only a few directly estimated from (~NLO) MC
  - Data driven and "hybrid" (data+MC)
    → large syst.uncertainties!
- No "golden" variable
  - Signal distributions and background distributions look similar




$H_T = \Sigma E_T$(lepton,Missing Energy,Jets)



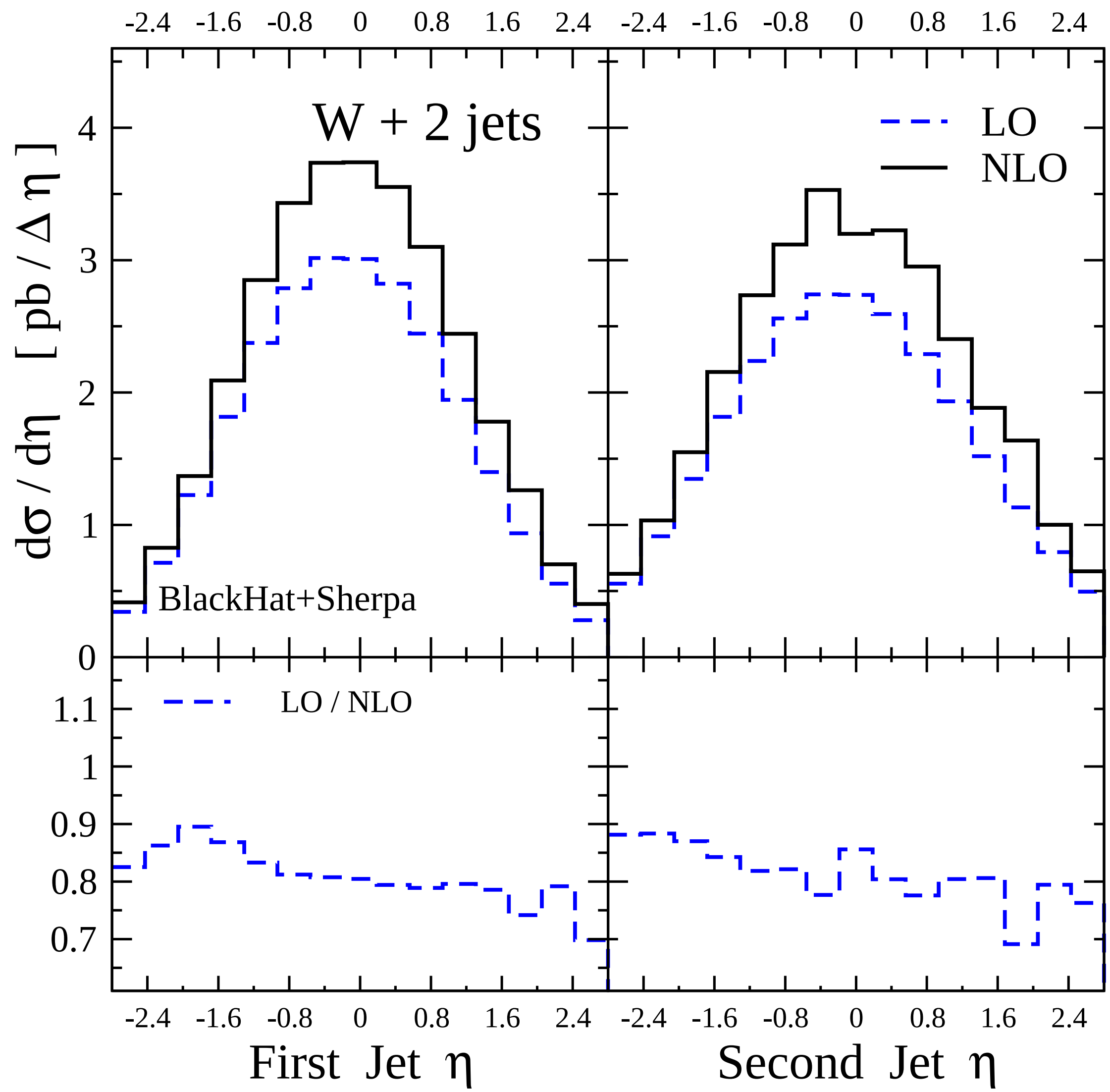




*March 10th, 2009* *Rainer Wallny - Observation of Electroweak Single Top Quark Production* 13/45

**From R. Wallny's (CDF) Fermilab Wine and Cheese 2009**

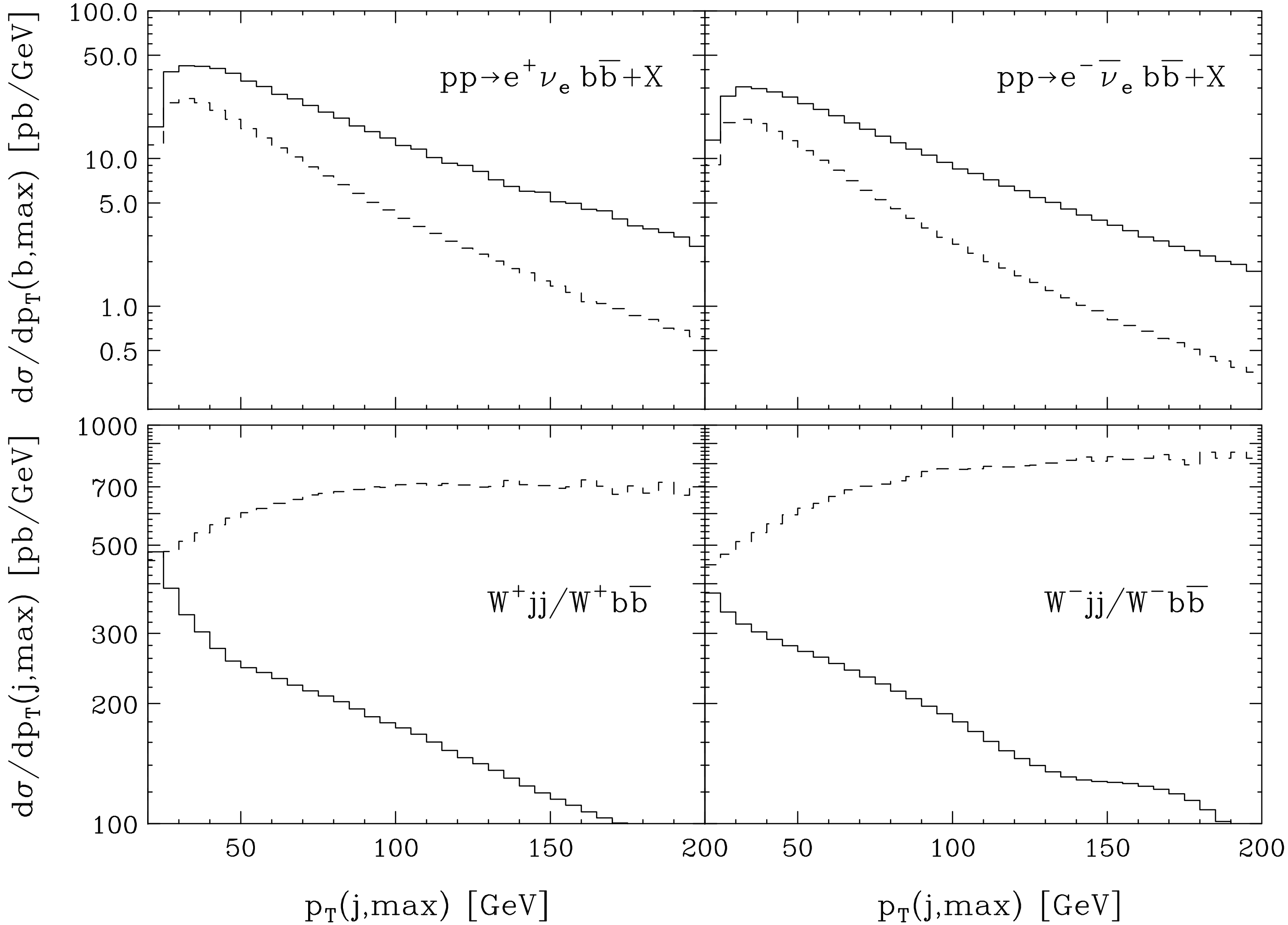
# Why NLO? Example: Single Top - Shape!!






CFB et al (BlackHat + SHERPA)

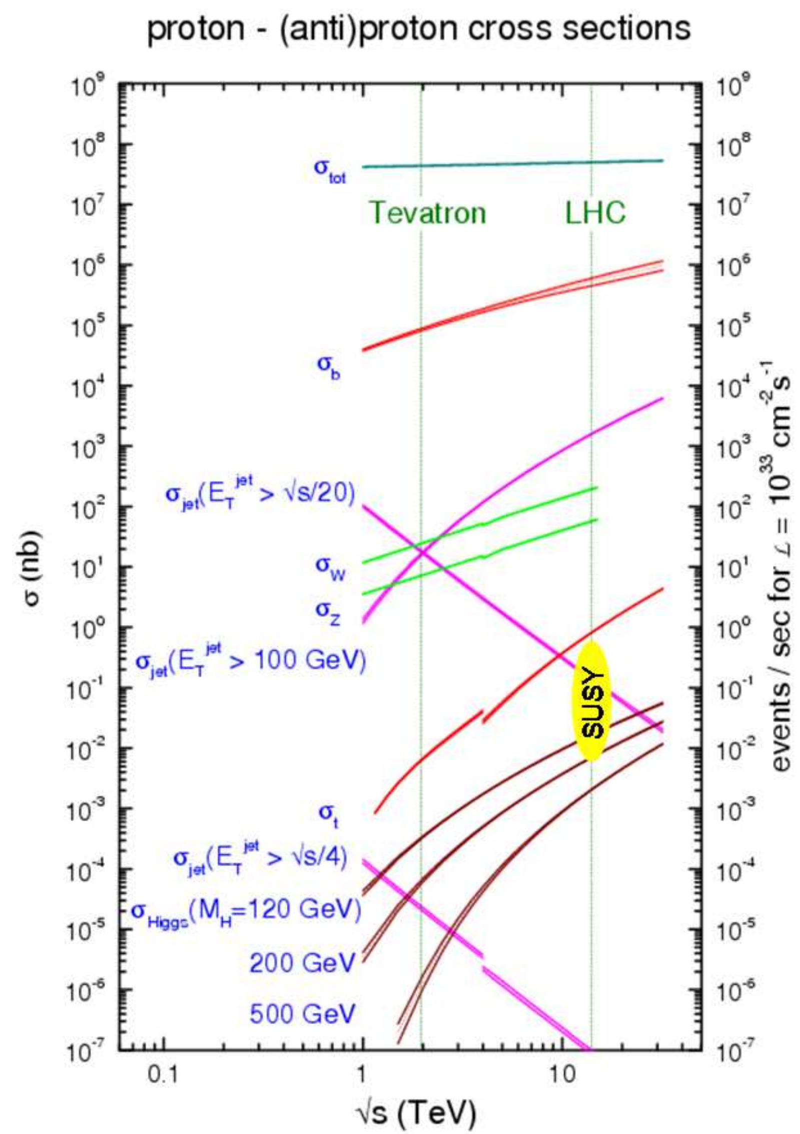
## Leading jet $p_T$ distribution of $Wb\bar{b}$ events
## LO dashed, NLO solid




Campbell, Ellis, Rainwater

# Why NLO?

## From the Tevatron to the LHC...




Campbell, Huston, Stirling

# Why NLO?



## From the Tevatron to the LHC...

# The (In)Famous Wishlist

BlackHat and Sherpa
It's 2010!
Outline

Introduction
Hadron Collisions
Why NLO? Example: Single Top
The LHC Wishlist
Why Not (Yet) NLO?
A Better Way?
NLO Corrections to LHC Processes

On-Shell Methods

Black Magic? BlackHat!

Summary and Outlook

Les Houches 2005

| process wanted at NLO ($V \in \{Z, W, \gamma\}$) | background to |
|---|---|
| 1. $pp \to VV +$ jet | $t\bar{t}H$, new physics |
| 2. $pp \to H + 2$ jets | $H$ production by vector boson fusion (VBF) |
| 3. $pp \to t\bar{t}b\bar{b}$ | $t\bar{t}H$ |
| 4. $pp \to t\bar{t} + 2$ jets | $t\bar{t}H$ |
| 5. $pp \to VVb\bar{b}$ | VBF $\to H \to VV$, $t\bar{t}H$, new physics |
| 6. $pp \to VV + 2$ jets | VBF $\to H \to VV$ |
| 7. $pp \to V + 3$ jets | new physics |
| 8. $pp \to VVV$ | SUSY trilepton |

# The (In)Famous Wishlist



Les Houches 2007

| process wanted at NLO ($V \in \{Z, W, \gamma\}$) | background to |
|---|---|
| 1. $pp \to VV +$ jet | $t\bar{t}H$, new physics |
| 2. $pp \to H + 2$ jets | $H$ production by vector boson fusion (VBF) |
| | $gg$: Campbell, Ellis, Zanderighi |
| 3. $pp \to t\bar{t}b\bar{b}$ | $t\bar{t}H$ |
| 4. $pp \to t\bar{t} + 2$ jets | $t\bar{t}H$ |
| 5. $pp \to VVb\bar{b}$ | VBF $\to H \to VV$, $t\bar{t}H$, new physics |
| 6. $pp \to VV + 2$ jets | VBF $\to H \to VV$ |
| | VBF: Bozzi, Jäger, Oleari, Zeppenfeld |
| 7. $pp \to V + 3$ jets | new physics |
| 8. $pp \to VVV$ | SUSY trilepton |
| | $ZZZ$: Lazopoulos, Melnikov, Petriello |
| 9. $pp \to b\bar{b}b\bar{b}$ | Higgs and new physics |

partially completed, via standard methods

# The (In)Famous Wishlist



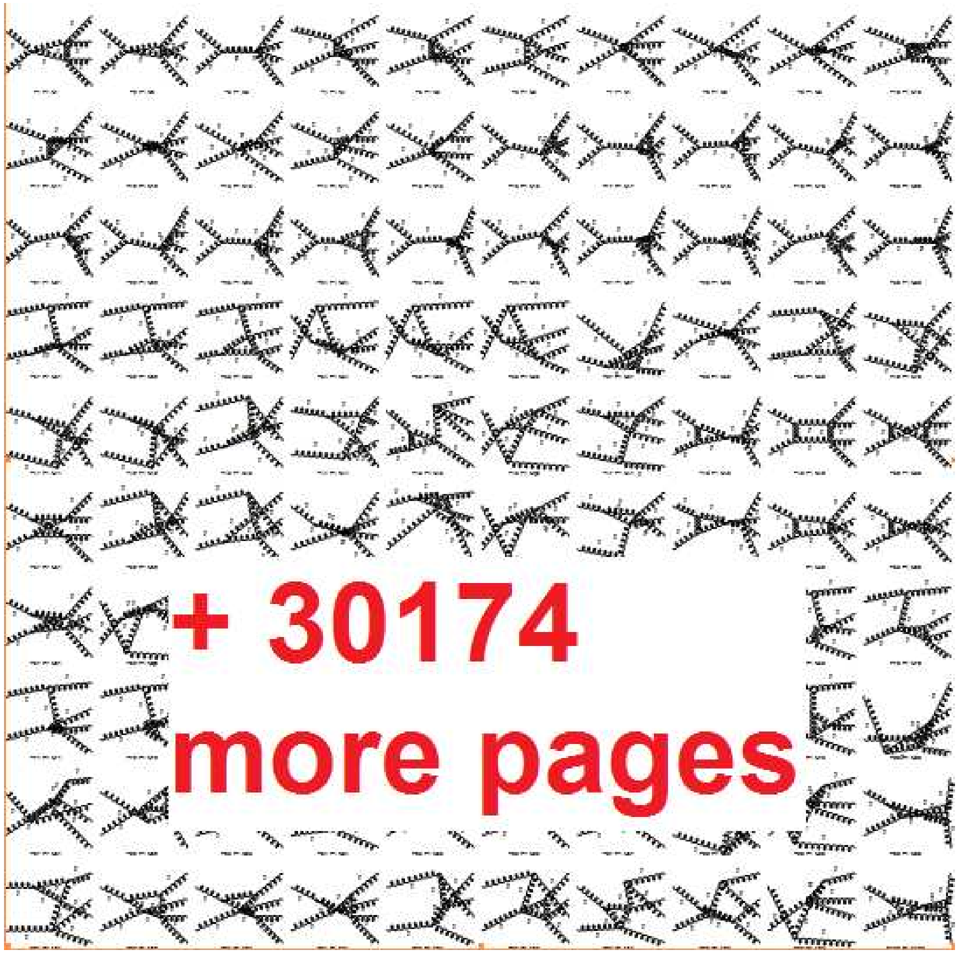
2010

| process wanted at NLO | background to |
|---|---|
| 1. $pp \to VV + $ jet | $t\bar{t}H$, new physics<br>Dittmaier, Kallweit, Uwer; Campbell, Ellis, Zanderighi |
| 2. $pp \to H + 2$ jets | $H$ in VBF<br>Campbell, Ellis, Zanderighi; Ciccolini, Denner Dittmaier |
| 3. $pp \to t\bar{t}b\bar{b}$ | $t\bar{t}H$ Bredenstein, Denner Dittmaier, Pozzorini;<br>Bevilacqua, Czakon, Papadopoulos, Pittau, Worek |
| 4. $pp \to t\bar{t} + 2$ jets | $t\bar{t}H$ Bevilacqua, Czakon, Papadopoulos, Worek |
| 5. $pp \to VVb\bar{b}$ | VBF $\to H \to VV$, $t\bar{t}H$, new physics |
| 6. $pp \to VV + 2$ jets | VBF $\to H \to VV$<br>VBF: Bozzi, Jäger, Oleari, Zeppenfeld |
| 7. $pp \to V + 3$ jets | new physics<br>CFB, Bern, Dixon, Febres Cordero, Forde, Gleisberg, Ita, Kosower, Maitre; Ellis, Melnikov, Zanderighi |
| 8. $pp \to VVV$ | SUSY trilepton<br>Lazopoulos, Melnikov, Petriello; Hankele, Zeppenfeld;<br>Binoth, Ossola, Papadopoulos, Pittau |
| 9. $pp \to b\bar{b}b\bar{b}$ | Higgs, new physics GOLEM |

# Why Not (Yet) NLO?



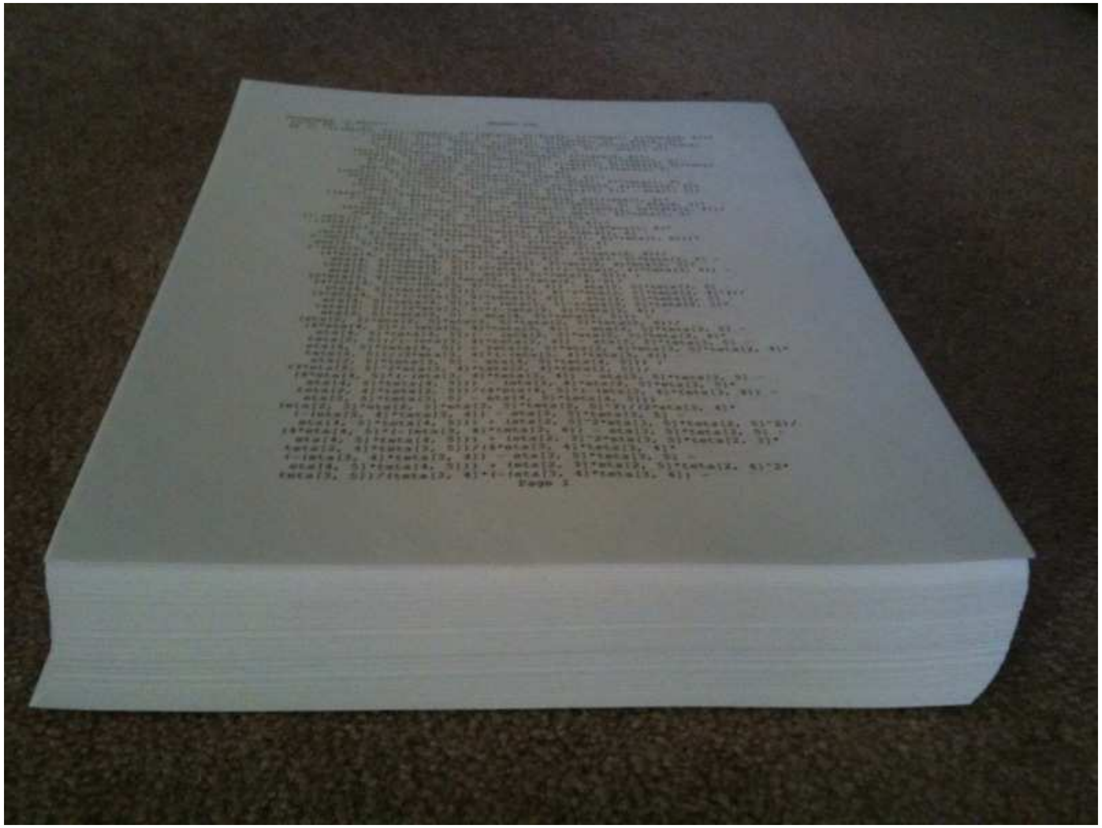
## One-loop 6-gluon Feynman diagrams:



# Why Not (Yet) NLO?



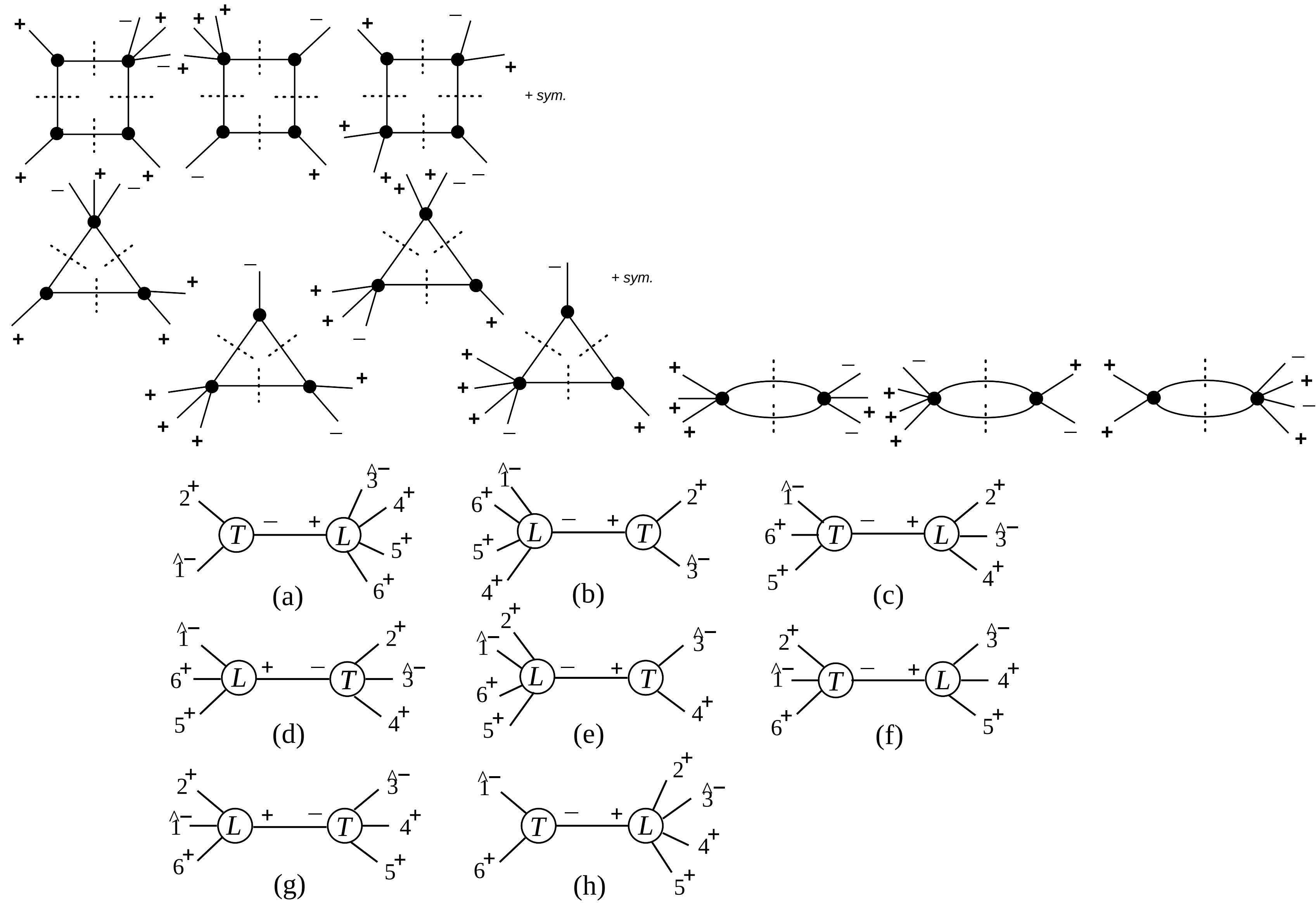
## Result for 1 helicity amplitude (rational part only):



Xiao, Yang, Zhu

# A Better Way?





# A Better Way?

## Result for the same helicity amplitude (rational part only):

$$
\begin{aligned}
&\widehat{R}_6^{a}(1,3)\\
&= i\Bigg[-\frac{[2\,4]^3}{3\langle 5\,6\rangle^2[1\,2][1\,3][3\,4]}-\frac{2\langle 1\,3\rangle^4}{9\langle 1\,2\rangle\langle 1\,6\rangle\langle 2\,3\rangle\langle 3\,4\rangle\langle 4\,5\rangle\langle 5\,6\rangle}\\
&+\frac{\langle 1\,3\rangle^2\langle 1\,4\rangle[2\,4]}{6\langle 1\,6\rangle\langle 2\,3\rangle\langle 2\,4\rangle\langle 4\,5\rangle\langle 5\,6\rangle[2\,3]}+\frac{\langle 1^-|(2+5)|4^-\rangle\langle 1\,3\rangle^2\langle 1\,5\rangle}{2\langle 1\,4\rangle\langle 1\,6\rangle\langle 2\,5\rangle^2\langle 3\,4\rangle\langle 5\,6\rangle[3\,4]}\\
&-\frac{\langle 1^-|(3+5)|4^-\rangle^3\langle 1^-|(4+5)|3^-\rangle[3\,5]}{3\langle 2^-|(4+5)|3^-\rangle\langle 6^-|(1+2)|3^-\rangle\langle 1\,2\rangle\langle 1\,6\rangle\langle 4\,5\rangle[3\,4]^2 s_{345}}\\
&-\frac{[2\,4]^3[3\,6]^3}{3\langle 5^-|(2+4)|3^-\rangle^2[1\,3][1\,6][2\,3][3\,4]}-\frac{\langle 1\,3\rangle^3\langle 1\,5\rangle[2\,5]}{6\langle 1^-|(3+4)|2^-\rangle\langle 1\,6\rangle\langle 2\,4\rangle\langle 2\,5\rangle\langle 3\,4\rangle\langle 5\,6\rangle}\\
&-\frac{\langle 1\,3\rangle^2\langle 1\,5\rangle(\langle 1^-|3(2+4)|5^+\rangle+4s_{24}\langle 1\,5\rangle)[2\,4][5\,6]}{6\langle 1^-|(2+3)|4^-\rangle\langle 1^-|(3+4)|2^-\rangle\langle 2\,4\rangle\langle 2\,5\rangle\langle 4\,5\rangle\langle 5\,6\rangle s_{234}}\\
&-\frac{s_{26}\langle 1^-|(4+5)|3^-\rangle^3[4\,5]^3}{3\langle 1^-|(3+4)|5^-\rangle\langle 2^-|4+5|3^-\rangle^2\langle 6^-|(1+2)|3^-\rangle^2[3\,4]s_{345}}\\
&-\frac{\langle 5^-|(1+3)|6^-\rangle[2\,6]s_{245}^2}{3\langle 4^-|(1+3)|6^-\rangle\langle 5^-|(2+4)|3^-\rangle\langle 2\,6\rangle\langle 4\,5\rangle\langle 5\,6\rangle[1\,3][1\,6]}\\
&-\frac{\langle 1\,3\rangle^3\langle 4\,6\rangle[2\,6]}{6\langle 4^-|(1+3)|2^-\rangle\langle 1\,2\rangle\langle 2\,6\rangle\langle 3\,4\rangle\langle 4\,5\rangle\langle 5\,6\rangle}+\frac{\langle 1^-|(2+3)|4^-\rangle^2\langle 1\,2\rangle\langle 1\,5\rangle[2\,4]}{2\langle 1^-|(2+4)|3^-\rangle\langle 1\,6\rangle\langle 2\,5\rangle^2\langle 5\,6\rangle[3\,4]s_{234}}\\
&+\frac{\langle 1\,3\rangle^3\langle 1\,5\rangle\langle 2\,5\rangle[4\,5][5\,6]}{6\langle 1^-|(2+3)|4^-\rangle\langle 2^-|(1+3)|6^-\rangle\langle 1\,6\rangle\langle 2\,3\rangle\langle 2\,4\rangle\langle 4\,5\rangle\langle 5\,6\rangle}\\
&+\frac{\langle 1^-|(3+5)|4^-\rangle(\langle 1\,5\rangle[5\,4]+\langle 1^-|(3+5)|4^-\rangle)\langle 1\,5\rangle^2[4\,5]}{6\langle 1\,2\rangle\langle 1\,6\rangle\langle 2\,5\rangle\langle 4\,5\rangle\langle 5\,6\rangle[3\,4]^2 s_{345}}\\
&-\frac{\langle 1^-|(3+5)|4^-\rangle^2\langle 1\,5\rangle^2[4\,5]}{2\langle 1^-|(4+5)|3^-\rangle\langle 1\,6\rangle\langle 2\,5\rangle^2\langle 5\,6\rangle[3\,4]s_{345}}-\frac{\langle 5^-|(1+3)|2^-\rangle^3\langle 4\,6\rangle[4\,5]}{3\langle 6^-|(1+2)|3^-\rangle\langle 4\,5\rangle^2\langle 5\,6\rangle^2[1\,2][1\,3]s_{123}}\\
&-\frac{(2\langle 1\,2\rangle[2\,4]+\langle 1\,3\rangle[3\,4])\langle 1^-|(2+3)|4^-\rangle\langle 1\,2\rangle[2\,4]}{6\langle 1\,6\rangle\langle 2\,4\rangle\langle 2\,5\rangle\langle 5\,6\rangle[3\,4]^2 s_{234}}\\
&+\frac{\langle 1\,3\rangle^2(\langle 1\,2\rangle^2\langle 4\,5\rangle[2\,4]-\langle 1\,5\rangle^2\langle 2\,4\rangle[4\,5])}{6\langle 1\,2\rangle\langle 1\,6\rangle\langle 2\,4\rangle\langle 2\,5\rangle\langle 3\,4\rangle\langle 4\,5\rangle\langle 5\,6\rangle[3\,4]}\\
&-\frac{(\langle 1\,3\rangle[3\,2]+2\langle 1\,4\rangle[4\,2])\langle 1^-|(3+4)|2^-\rangle\langle 1\,4\rangle[2\,4]}{6\langle 1\,6\rangle\langle 2\,4\rangle\langle 4\,5\rangle\langle 5\,6\rangle[2\,3]^2 s_{234}}\\
&+\frac{(s_{45}+s_{56})\langle 1\,3\rangle^3[4\,6]^2}{3\langle 1^-|(2+3)|4^-\rangle\langle 2^-|(1+3)|6^-\rangle\langle 2\,3\rangle\langle 4\,5\rangle\langle 5\,6\rangle s_{123}}\\
&-\frac{\langle 1^-|(2+4)|3^-\rangle^3[2\,4]^3}{3\langle 5^-|(2+4)|3^-\rangle\langle 1\,6\rangle\langle 2\,4\rangle\langle 5\,6\rangle[2\,3]^2[3\,4]^2 s_{234}}\\
&+\frac{\langle 6^-|(1+3)|2^-\rangle^2[2\,6]s_{123}}{3\langle 4^-|(1+3)|2^-\rangle\langle 2\,6\rangle^2\langle 4\,5\rangle\langle 5\,6\rangle[1\,2][1\,3][2\,3]}\\
&+\frac{\langle 5^-|(1+3)|2^-\rangle\langle 6^-|(1+3)|2^-\rangle^2[2\,6]}{3\langle 4^-|(1+3)|2^-\rangle\langle 6^-|(1+2)|3^-\rangle\langle 4\,5\rangle\langle 5\,6\rangle^2[1\,2][1\,3]}\\
&-\frac{(s_{12}+s_{23})\langle 6^-|(1+3)|2^-\rangle\langle 1\,6\rangle^2[2\,6]s_{123}}{6\langle 4^-|(1+3)|2^-\rangle\langle 6^-|(1+2)|3^-\rangle\langle 1\,2\rangle\langle 2\,6\rangle^2\langle 4\,5\rangle\langle 5\,6\rangle[1\,2][2\,3]}\\
&-\frac{\langle 1^-|(5+6)(2+4)|5^+\rangle^2\langle 1\,5\rangle^2[2\,4][5\,6]}{2\langle 1^-|(2+4)|3^-\rangle\langle 1^-|(3+4)|2^-\rangle\langle 5^-|(2+4)|3^-\rangle\langle 2\,5\rangle^2\langle 4\,5\rangle\langle 5\,6\rangle s_{234}}\\
&+\frac{\langle 1^-|(3+4)|2^-\rangle\langle 5^-|(1+3)|6^-\rangle\langle 1\,5\rangle[3\,6]s_{245}^2}{3\langle 2^-|(4+5)|3^-\rangle\langle 4^-|(1+3)|6^-\rangle\langle 5^-|(2+4)|3^-\rangle^2\langle 1\,6\rangle\langle 4\,5\rangle\langle 5\,6\rangle[1\,6]}\\
&+\frac{\langle 5^-|(1+3)|6^-\rangle\langle 1\,3\rangle[2\,5][3\,6](\langle 5^-|(2+4)|3^-\rangle\langle 1\,3\rangle+2\langle 1\,5\rangle s_{245})}{3\langle 2^-|(4+5)|3^-\rangle\langle 4^-|(1+3)|6^-\rangle\langle 5^-|(2+4)|3^-\rangle\langle 1\,6\rangle\langle 4\,5\rangle\langle 5\,6\rangle[1\,6]}
\end{aligned}
$$

$$
\begin{aligned}
&-\frac{\langle 1^-|(2+6)(4+5)|6^+\rangle^2\langle 1\,2\rangle\langle 1\,6\rangle[2\,6][4\,5]}{2\langle 1^-|(3+4)|5^-\rangle\langle 1^-|(4+5)|3^-\rangle\langle 6^-|(1+2)|3^-\rangle\langle 2\,6\rangle^2\langle 4\,5\rangle\langle 5\,6\rangle s_{345}}\\
&+\frac{\langle 1^-|(2+6)(4+5)|6^+\rangle\langle 1\,6\rangle(\langle 1^-|3(4+5)|6^+\rangle+2s_{45}\langle 1\,6\rangle)[2\,6][4\,5]}{6\langle 1^-|(3+4)|5^-\rangle\langle 6^-|(1+2)|3^-\rangle^2\langle 2\,6\rangle\langle 4\,5\rangle\langle 5\,6\rangle s_{345}}\\
&-\frac{\langle 1^-|(2+6)(4+5)|2^+\rangle\langle 1\,2\rangle(\langle 1^-|3(4+5)|2^+\rangle+2s_{45}\langle 1\,2\rangle)[2\,6][4\,5]}{6\langle 1^-|(3+4)|5^-\rangle\langle 2^-|(4+5)|3^-\rangle^2\langle 2\,5\rangle\langle 2\,6\rangle\langle 4\,5\rangle s_{345}}\\
&+\frac{\langle 1^-|(2+3)|4^-\rangle\langle 1^-|(3+4)|2^-\rangle[2\,4](-\langle 1\,2\rangle^2\langle 4\,5\rangle[2\,3]+\langle 1\,4\rangle^2\langle 2\,5\rangle[3\,4])}{2\langle 1^-|(2+4)|3^-\rangle\langle 1\,6\rangle\langle 2\,4\rangle\langle 2\,5\rangle\langle 4\,5\rangle\langle 5\,6\rangle[2\,3][3\,4]s_{234}}\\
&+\frac{\langle 1\,3\rangle^3\langle 4\,6\rangle^2[2\,6](2\langle 1^-|2(5+6)|4^+\rangle+\langle 1^-|65|4^+\rangle+\langle 1\,4\rangle\langle 2\,6\rangle[2\,6])}{2\langle 1^-|(3+4)|5^-\rangle\langle 4^-|(1+3)|2^-\rangle\langle 1\,4\rangle\langle 2\,6\rangle^2\langle 3\,4\rangle\langle 4\,5\rangle^2\langle 5\,6\rangle}\\
&-\frac{\langle 5^-|(1+3)|6^-\rangle^2\langle 1\,5\rangle(3\langle 2^-|(4+5)|6^-\rangle\langle 1\,5\rangle-\langle 1\,3\rangle\langle 2\,5\rangle[3\,6])[5\,6]s_{245}}{6\langle 2^-|(4+5)|6^-\rangle\langle 4^-|(1+3)|6^-\rangle\langle 5^-|(2+4)|3^-\rangle\langle 1\,6\rangle\langle 2\,5\rangle^2\langle 4\,5\rangle\langle 5\,6\rangle[1\,6][3\,6]}\\
&-\frac{\langle 1\,3\rangle^3\langle 1\,5\rangle(\langle 1\,4\rangle\langle 1\,5\rangle\langle 2\,4\rangle\langle 2\,5\rangle[2\,5]^2-\langle 1^-|52|4^+\rangle\langle 1\,6\rangle\langle 4\,5\rangle[5\,6])}{2\langle 1^-|(3+4)|2^-\rangle\langle 4^-|(1+3)|6^-\rangle\langle 1\,4\rangle\langle 1\,6\rangle^2\langle 2\,4\rangle\langle 2\,5\rangle^2\langle 3\,4\rangle\langle 5\,6\rangle}\\
&+\frac{\langle 1^-|(2+6)(4+5)|2^+\rangle\langle 1\,2\rangle[2\,6][4\,5]}{2\langle 1^-|(3+4)|5^-\rangle\langle 1^-|(4+5)|3^-\rangle\langle 2^-|(4+5)|3^-\rangle\langle 2\,5\rangle^2\langle 2\,6\rangle^2\langle 4\,5\rangle s_{345}}\\
&\quad\times(\langle 1^-|(2+6)(4+5)|2^+\rangle\langle 1\,6\rangle\langle 2\,5\rangle+\langle 1^-|(2+6)4|5^+\rangle\langle 1\,2\rangle\langle 2\,6\rangle)\\
&-\frac{\langle 5^-|(1+3)|6^-\rangle^3}{3\langle 2^-|(4+5)|3^-\rangle\langle 2^-|(4+5)|6^-\rangle\langle 5^-|(2+4)|3^-\rangle\langle 4\,5\rangle\langle 5\,6\rangle[1\,6]}\\
&\quad\times\left(\frac{\langle 4\,6\rangle[3\,6][4\,5]}{\langle 4\,5\rangle[1\,3]}+\frac{\langle 1\,2\rangle[5\,6]s_{245}}{\langle 4^-|(1+3)|6^-\rangle\langle 2\,5\rangle}\right)\\
&+\frac{\langle 1\,3\rangle^3(\langle 1\,5\rangle\langle 2\,6\rangle\langle 4\,5\rangle[2\,5](6\langle 1\,5\rangle\langle 2\,4\rangle+\langle 1\,4\rangle\langle 2\,5\rangle)+2\langle 1\,4\rangle\langle 1\,6\rangle\langle 2\,5\rangle^2\langle 4\,6\rangle[2\,6])}{6\langle 4^-|(1+3)|6^-\rangle\langle 1\,4\rangle\langle 1\,6\rangle^2\langle 2\,5\rangle^2\langle 2\,6\rangle\langle 3\,4\rangle\langle 4\,5\rangle\langle 5\,6\rangle}\\
&-\frac{\langle 1\,5\rangle^3[2\,4]^2[5\,6]}{6\langle 1^-|(2+3)|4^-\rangle\langle 1^-|(3+4)|2^-\rangle\langle 5^-|(2+4)|3^-\rangle\langle 2\,5\rangle\langle 4\,5\rangle\langle 5\,6\rangle^2[2\,3][3\,4]s_{234}}\\
&\quad\times\Big(2\langle 6\,4\rangle[4\,3]\langle 1\,4\rangle\langle 2\,5\rangle[2\,4]^2+2\langle 6\,2\rangle[2\,3]\langle 1\,2\rangle\langle 4\,5\rangle[2\,4]^2-5s_{24}\langle 1\,3\rangle\langle 5\,6\rangle[2\,3][3\,4]\Big)\\
&+\frac{\langle 2^-|(1+3)|6^-\rangle\langle 1\,2\rangle[2\,6]s_{245}}{6\langle 2^-|(4+5)|3^-\rangle\langle 4^-|(1+3)|6^-\rangle\langle 1\,6\rangle\langle 2\,5\rangle^2\langle 2\,6\rangle^2\langle 4\,5\rangle\langle 5\,6\rangle[1\,6][3\,6]}\\
&\quad\times\Big(3\langle 6^-|(1+3)|6^-\rangle\langle 1\,6\rangle\langle 2\,5\rangle^2-3\langle 5^-|(1+3)|6^-\rangle\langle 1\,5\rangle\langle 2\,6\rangle^2\\
&\qquad-\langle 1\,3\rangle\langle 2\,5\rangle\langle 2\,6\rangle\langle 5\,6\rangle[3\,6]\Big)\\
&-\frac{\langle 1\,3\rangle^3}{3\langle 1^-|(3+4)|5^-\rangle\langle 4^-|(1+3)|6^-\rangle\langle 3\,4\rangle\langle 5\,6\rangle}\Bigg(\frac{\langle 1\,2\rangle\langle 1\,5\rangle^2[2\,5]^3}{\langle 1^-|(3+4)|2^-\rangle\langle 1\,6\rangle^2\langle 2\,5\rangle}\\
&\qquad-\frac{\langle 1\,4\rangle[2\,5]^2[2\,6]^2}{\langle 1^-|(3+4)|2^-\rangle\langle 4^-|(1+3)|2^-\rangle}+\frac{\langle 2\,4\rangle\langle 4\,6\rangle^2[2\,6]^3}{\langle 4^-|(1+3)|2^-\rangle\langle 2\,6\rangle\langle 4\,5\rangle^2}\Bigg)\\
&-\frac{[2\,6]s_{245}^2}{3\langle 2^-|(4+5)|3^-\rangle\langle 4^-|(1+3)|6^-\rangle\langle 5^-|(2+4)|3^-\rangle\langle 1\,6\rangle\langle 2\,5\rangle\langle 2\,6\rangle\langle 4\,5\rangle\langle 5\,6\rangle[1\,3][1\,6]}\\
&\quad\times\Big(\langle 5^-|(1+3)|6^-\rangle\langle 1\,2\rangle[1\,3](\langle 1\,2\rangle\langle 5\,6\rangle-\langle 1\,6\rangle\langle 2\,5\rangle)\\
&\quad-\langle 1^-|(3+6)(1+3)|2^+\rangle\langle 2\,5\rangle\langle 5\,6\rangle[3\,6]\Big)\\
&+\frac{\langle 1\,3\rangle^3}{6\langle 1^-|(3+4)|5^-\rangle\langle 1\,4\rangle\langle 1\,6\rangle\langle 2\,5\rangle^2\langle 2\,6\rangle^2\langle 3\,4\rangle\langle 4\,5\rangle^2\langle 5\,6\rangle}\\
&\quad\times\Big(6\langle 1\,5\rangle^2\langle 2\,4\rangle\langle 2\,6\rangle^2\langle 4\,5\rangle[2\,5]-2\langle 1\,4\rangle\langle 1\,5\rangle\langle 2\,5\rangle\langle 2\,6\rangle^2\langle 4\,5\rangle[2\,5]\\
&\quad-6\langle 1\,6\rangle^2\langle 2\,4\rangle\langle 2\,5\rangle^2\langle 4\,6\rangle[2\,6]+5\langle 1\,4\rangle\langle 1\,6\rangle\langle 2\,5\rangle^2\langle 2\,6\rangle\langle 4\,6\rangle[2\,6]\Big)\\
&+\frac{\langle 1\,2\rangle\langle 1\,3\rangle^3\langle 1\,5\rangle^2\langle 2\,4\rangle[2\,5]^2}{\langle 1^-|(3+4)|5^-\rangle\langle 4^-|(1+3)|6^-\rangle\langle 1\,4\rangle\langle 1\,6\rangle^2\langle 2\,5\rangle^2\langle 3\,4\rangle\langle 5\,6\rangle}\\
&-\frac{\langle 1\,2\rangle\langle 1\,3\rangle^3\langle 2\,4\rangle^2[2\,5][2\,6]}{2\langle 1^-|(3+4)|5^-\rangle\langle 4^-|(1+3)|6^-\rangle\langle 1\,4\rangle\langle 2\,5\rangle\langle 2\,6\rangle^2\langle 3\,4\rangle\langle 4\,5\rangle}\\
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\end{aligned}
$$

**CFB, Bern, Dixon, Forde, Kosower**

# NLO Corrections to LHC Processes



- **Relevant processes all $2 \to n \geq 3$ as listed in the experimenters' (in)famous Les Houches wishlist**

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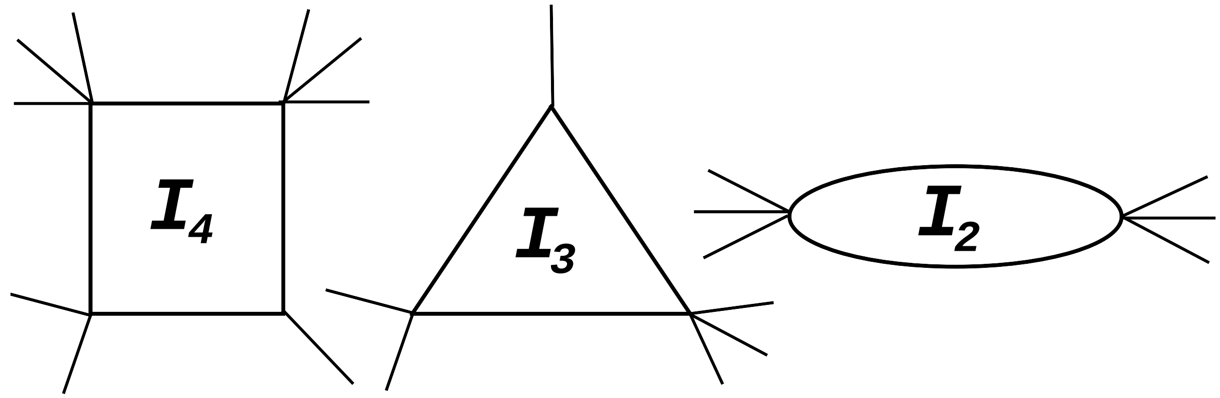
# On-Shell Methods



# One-Loop Decomposition




Any $n$-leg (massless) one-loop amplitude expressible in terms of scalar box, triangle and bubble integrals:

$$\mathcal{A} = c_4 I_4 + c_3 I_3 + c_2 I_2 + \text{ rational}$$

With massive partons there are additionally $I_1$ (tadpoles)

We know the integrals, the task is to determine the coefficients

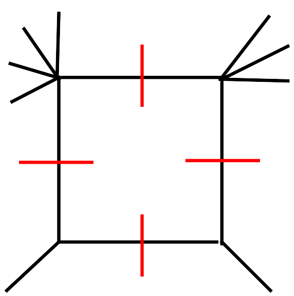
Bern, Dixon, Dunbar, Kosower

# Generalized Unitarity

$$c_4 I_4 = c_4 \int d^4 l \frac{1}{l^2 (l-K_1)^2 (l-K_2)^2 (l-K_3)^2}$$

$$\frac{1}{P^2 + i\varepsilon} = \frac{1}{P^2} + i\delta^+(P^2)$$

**Box integrals have unique leading singularity $\Rightarrow$ generalized unitarity**



$$c_4 \Delta_{LS} I_4 = \int d^4 l \delta^+(l^2) \delta^+((l-K_1)^2)$$
$$\times \delta^+((l-K_2)^2) \delta^+((l-K_3)^2)$$
$$\times A_1^{\text{tree}}(l) \times A_2^{\text{tree}}(l) \times A_3^{\text{tree}}(l) \times A_4^{\text{tree}}(l)$$

$$c_4 = A_1^{\text{tree}}(l_{\text{sol}}) \times A_2^{\text{tree}}(l_{\text{sol}}) \times A_3^{\text{tree}}(l_{\text{sol}}) \times A_4^{\text{tree}}(l_{\text{sol}})$$

**Tree graphs on shell**

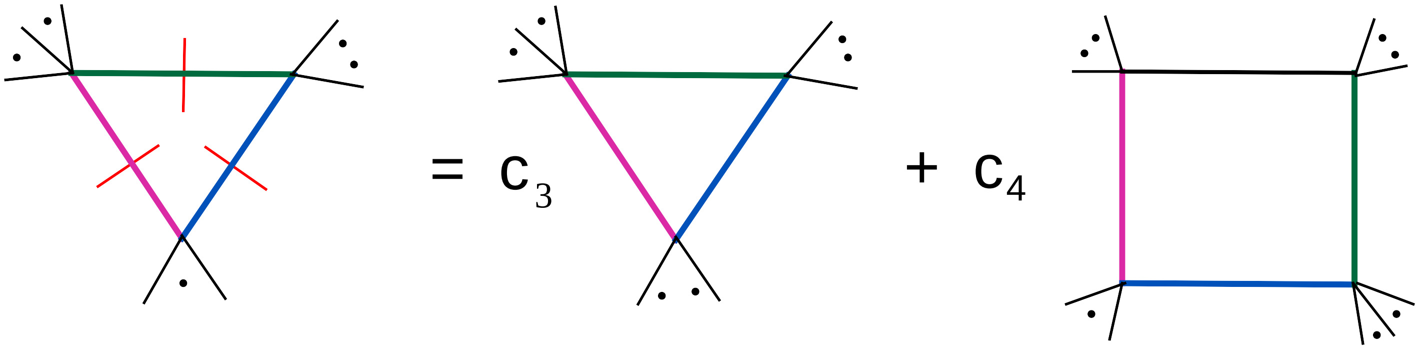
**Trees "recycled" into loops**

Britto, Cachazo, Feng

# Generalized Unitarity contd.



**Triangle coefficients from triple cuts, bubble coefficients from double cuts.**



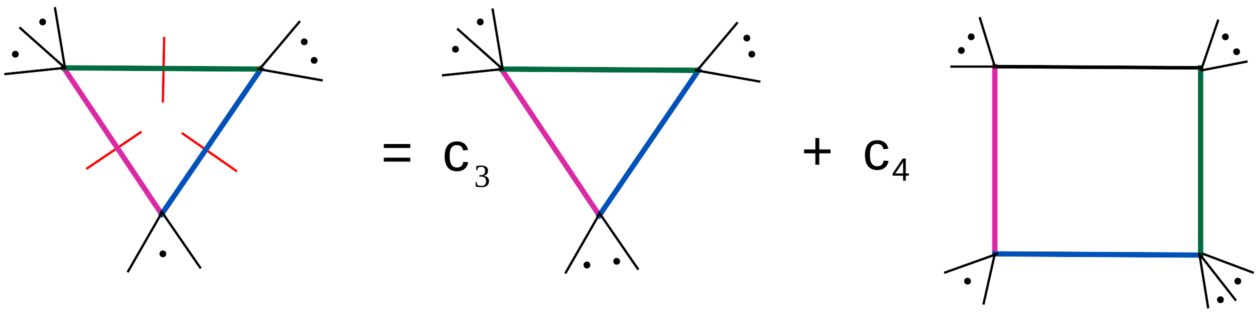

**But life's not so simple – "leakage" from higher-point integrals into lower point ones because integrals are not fully localized any more.**

**However, the singularity structures are unique – need procedure to disentangle coefficients: Clever parametrization of integral – read off coefficients directly** Forde; Ossola, Papadopoulos, Pittau; Kilgore

# Disentangling Coefficients

**Parametrization of loop momenta (schematically):**

$$l_n^\mu = \alpha_1 K_1^\mu + \alpha_2 K_2^\mu + \alpha_3 t K_3(K_1, K_2)^\mu + \frac{\alpha_4}{t} K_4(K_1, K_2)^\mu$$




**Triple cut then gives:**

$$C_3 = \sum_{j=-3}^{3} c_j t^j + \sum_i \frac{b_i}{\xi_i (t - t_i)}$$

$l_i^2(t) \sim \xi_i (t - t_i)$.

**Boxes have extra poles in $t$ from propagators that go on-shell.**

**But we know the boxes, so subtract them off.**

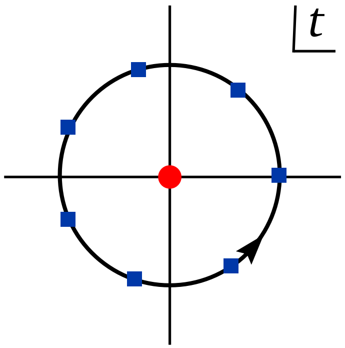
# Disentangling Coefficients contd.

**Triangle contributions after subtraction of boxes:**

$$T_3 = \sum_{j=-3}^{3} c_j t^j$$

**$c_0$ is the triangle coefficient, extract via discrete Fourier transform**

$$c_0 = \frac{1}{7} \sum_{j=0}^{6} T_3 \left( t_0 e^{2\pi i j/7} \right)$$

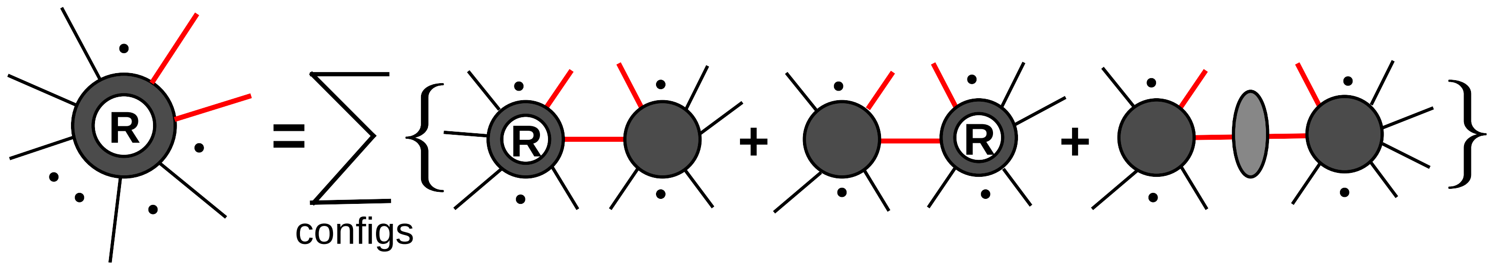



**BlackHat: CFB, Bern, Dixon, Febres-Cordero, Forde, Ita, Kosower, Maitre**

# Rational Terms from Recursion



$$\mathcal{A} = \sum_i c_i I_i + \text{ rational}$$



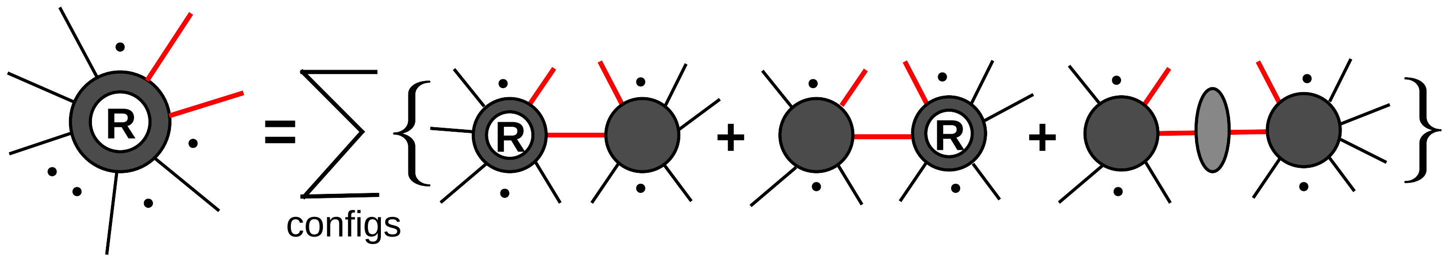

$$R = \sum_{\text{configs}} A_L \frac{1}{P_{l\ldots m}^2} A_R$$

**Not as simple as the analogous (BCFW) tree level recursion,**

CFB, Bern, Dixon, Forde, Kosower

**Alternative approach (also in BlackHat):**
**$D$-dimensional unitarity**

# Recursion at Loop Level




**Complex continue amplitude**

$$A(z) = C(z) + R(z) \qquad \Bigg| \quad \frac{1}{2\pi i}\oint_C \frac{dz}{z}$$

$$A(0) = C(0) - \sum_{\text{poles } \alpha} \operatorname*{Res}_{z=z_\alpha} \frac{R(z)}{z}$$

$$= C(0) + \sum_{\text{configs}} A_L \frac{1}{P^2_{l\ldots m}} A_R$$

**Loops “recycled” into loops**

CFB, Bern, Dixon, Forde, Kosower

**(ignoring several subtleties)**

# Rational Terms - D-dim Unitarity



**Unitarity in $D = 4 - 2\varepsilon$:**
**Split up into 4-D piece and $(-2\varepsilon)$-dim. piece (∼ small "mass")**

$$l_D^2 = l_4^2 + l_{[-2\varepsilon]}^2 = l_4^2 + \mu^2$$

$$\int \frac{d^D l}{(2\pi)^D} = \int \frac{d^4 l_4}{(2\pi)^4} \int \frac{d^{-\varepsilon}(\mu^2)}{(2\pi)^{-2\varepsilon}}$$

**Extract rational part $R$ by keeping track of $\mu$-dependence in generalized unitarity cuts:**

$$\mathcal{A} = c_4^{[0]} I_4^D[1] + c_4^{[2]} I_4^D[\mu^2] + c_4^{[4]} I_4^D[\mu^4] + c_3^{[0]} I_3^D[1] + \ldots$$

$$I_n^D[\mu^{2r}] = \frac{1}{2^r} I_n^{D+2r}[1] \prod_{k=0}^{r-1} (D - 4 + k)$$

# Rational Terms - D-dim Unitarity contd.

$$\mathcal{A} = c_4^{[0]} I_4^D[1] + c_4^{[2]} I_4^D[\mu^2] + c_4^{[4]} I_4^D[\mu^4] + c_3^{[0]} I_3^D[1] + \ldots$$

$$\begin{aligned}\mathcal{A} &= c_4^{[0]} I_4^D + \frac{D-4}{2} c_4^{[2]} I_4^{D+2} \\ &\quad + \frac{(D-4)(D-2)}{4} c_4^{[4]} I_4^{D+4} + c_3^{[0]} I_3^D + \ldots \\ &= c_4^{[0]} I_4^{4-2\varepsilon} + c_3^{[0]} I_3^{4-2\varepsilon} + c_2^{[0]} I_2^{4-2\varepsilon} + R\end{aligned}$$

$$R = c_4^{[4]}\, I_4^{4-2\varepsilon}[\mu^4]\Big|_{\varepsilon=0} + c_3^{[2]}\, I_3^{4-2\varepsilon}[\mu^4]\Big|_{\varepsilon=0} + \ldots$$

**Badger, Forde. See also Ossola, Papadopoulos, Pittau (CutTools); Ellis, Giele, Kunszt, Melnikov, Zanderighi (Rocket).**

# BlackHat



$$\mathcal{A} = \sum_i c_i I_i + \text{ rational}$$

- **Cut parts from 4-D unitarity**

# BlackHat



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- Rational parts from loop recursion

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- **OR rational parts from D-dim unitarity** $\Rightarrow$ **4-D unitarity with small "mass"**

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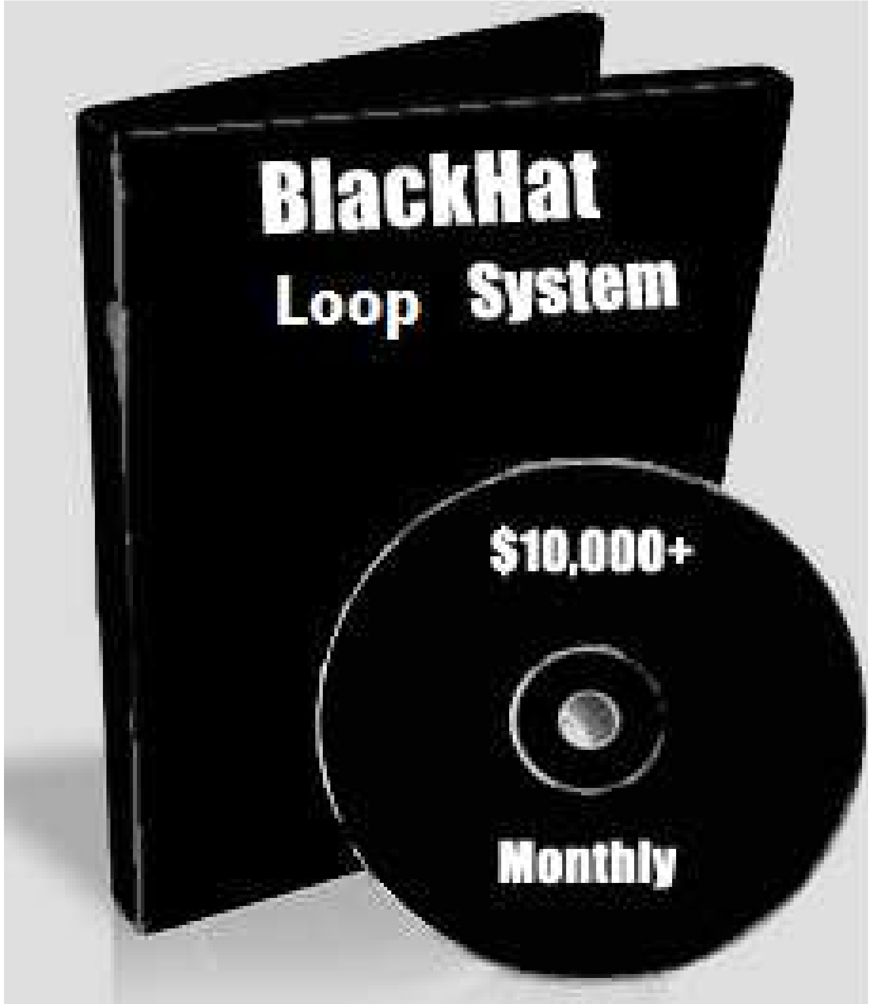
$\Rightarrow$ summary in review: CFB, Forde, arXiv:0912.3534 (ARNPS)

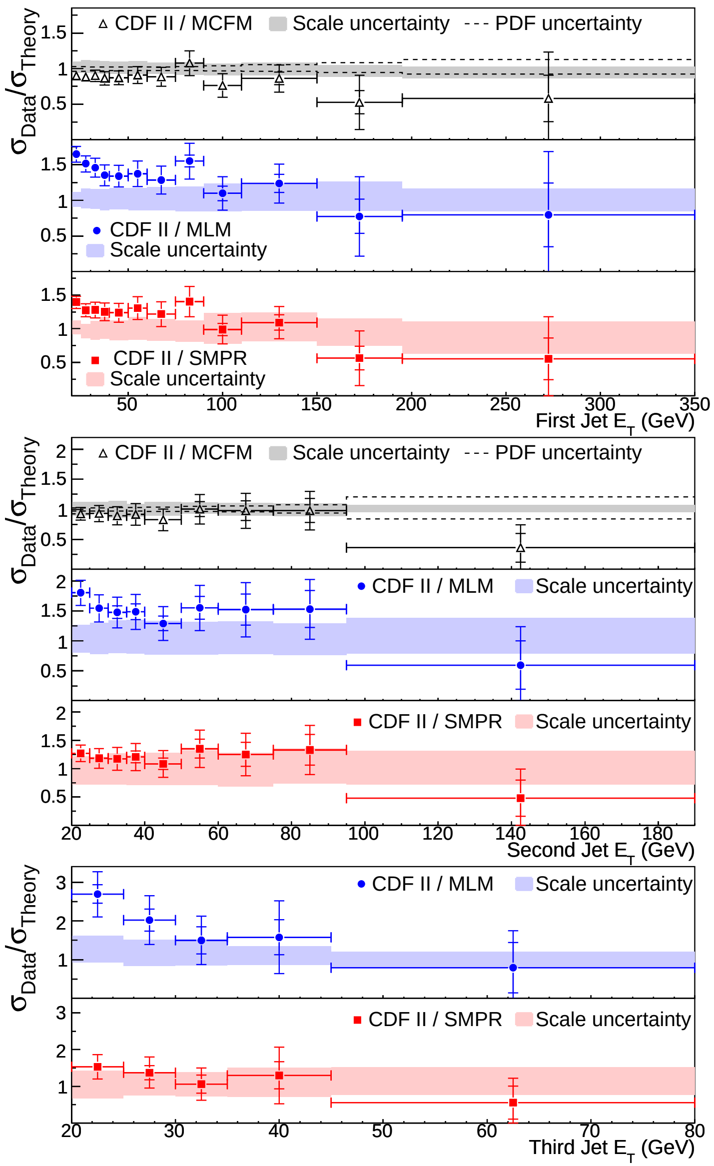
# Black Magic? BlackHat! – Results





# $W + 1, 2, 3$ jets at the Tevatron



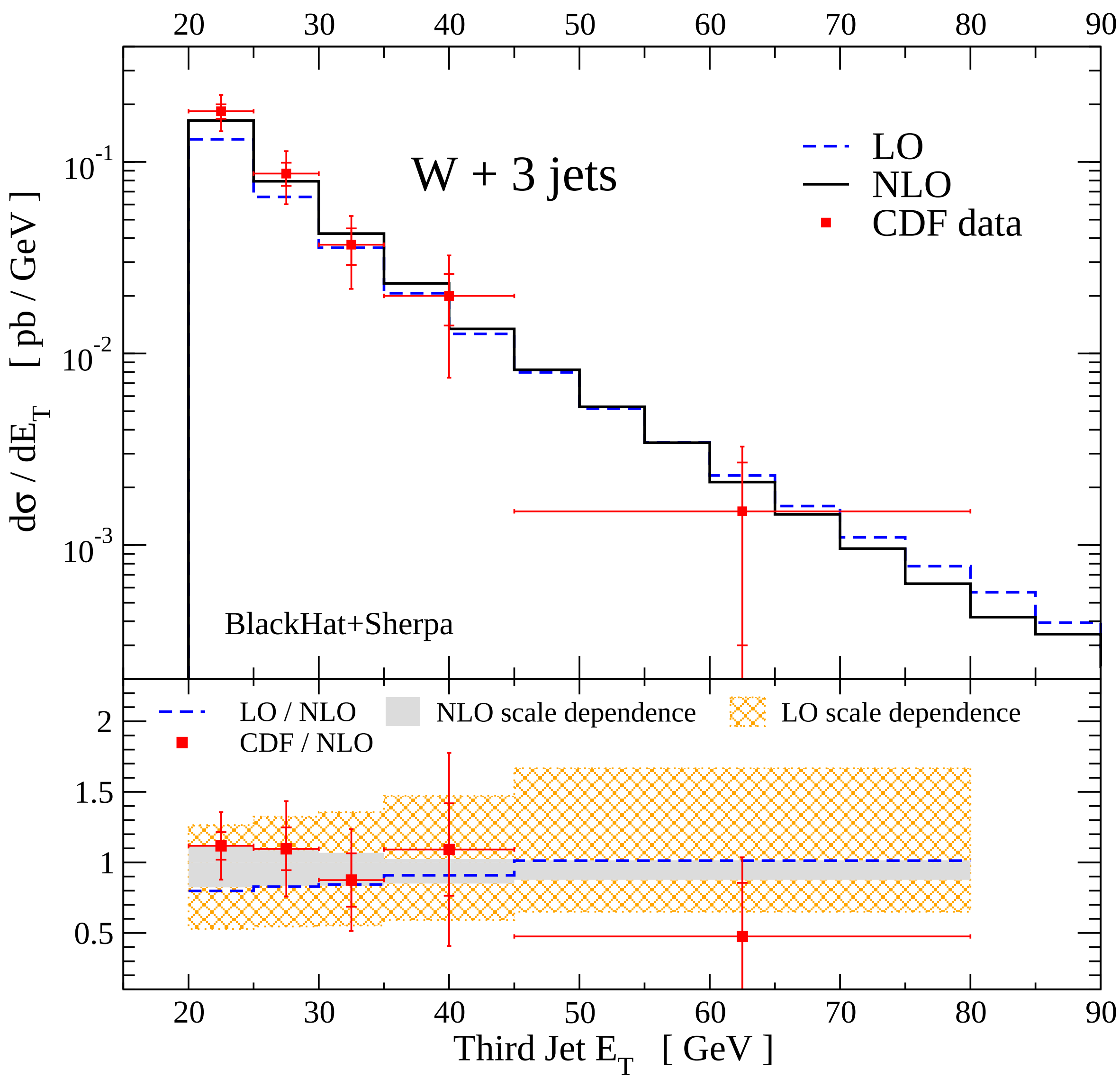



**No MCFM for W + 3 jets**

CDF 2007

# $W + 3$ jets at the Tevatron – BlackHat






**BlackHat + Gleisberg (Sherpa - for real emissions)**

# $W$s and $Z$s



- $W(\rightarrow l\nu)+$ jets and $Z(\rightarrow \nu\nu)+$ jets significant (irreducible) backgrounds in searches for new physics
- $Z \rightarrow \nu\nu$-background can be calibrated ("data-driven") from $Z \rightarrow l^+l^-$, but especially in initial LHC running needs to be supplemented by theory input (Monte Carlos,...) because of very low statistics
- $W \rightarrow l\nu$ has larger cross section, but is less clean because of intrinsic missing energy
- Underlying QCD dynamics is the same for both $\Rightarrow$ can use $Z/W$ to calibrate $W/Z$
- ATLAS and CMS estimate uncertainty up to 30-50% due to (LO) Monte Carlo input

# $W$ + 3 Jets at the LHC

- BlackHat and Sherpa
- It's 2010!
- Outline

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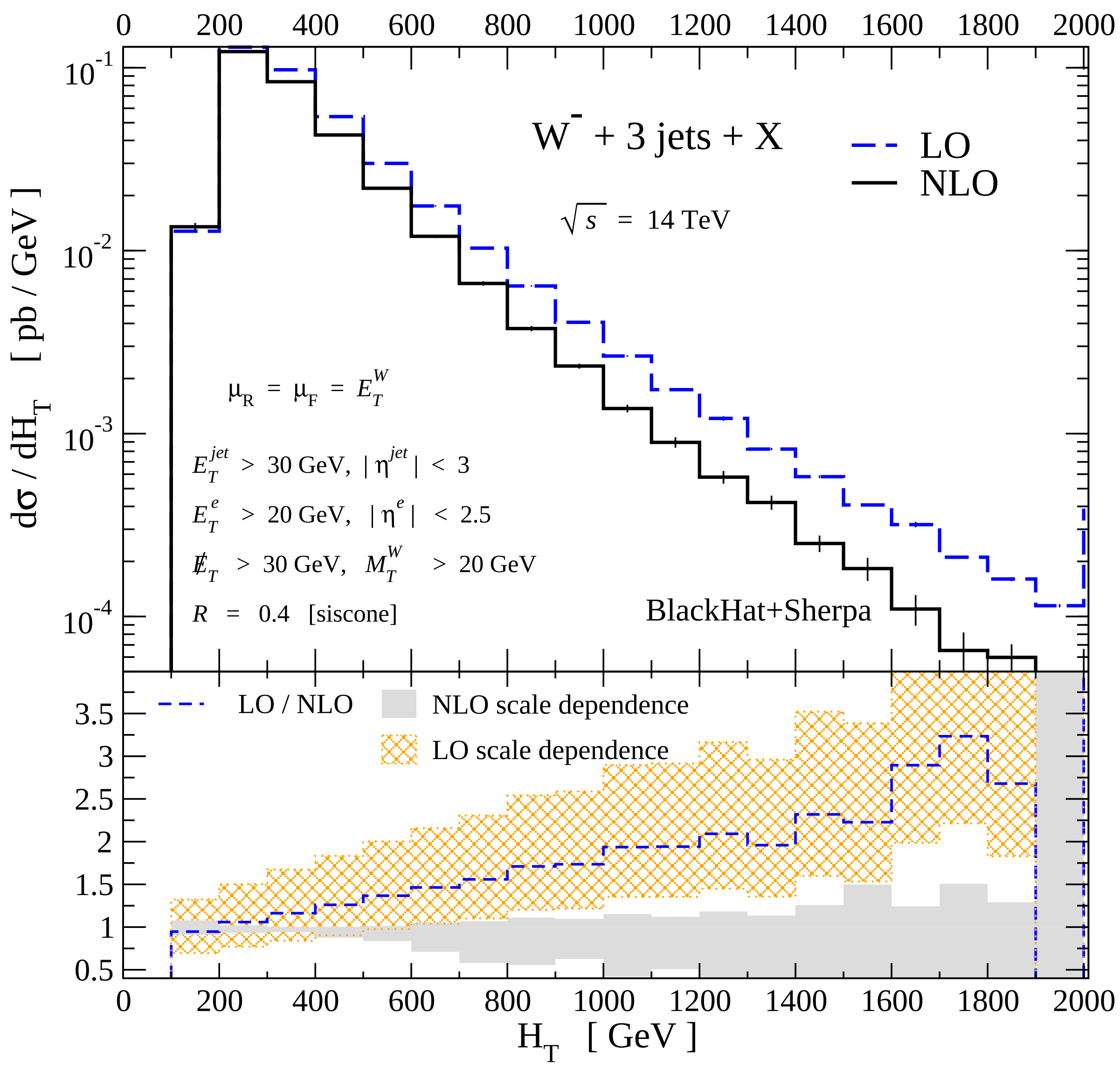
- Tevatron
- $W$s and $Z$s
- $W$ + 3 Jets at the LHC
- $Z$ + 3 Jets at the Tevatron
- Towards $W$ + 4 Jets
- More to Come!

Summary and Outlook



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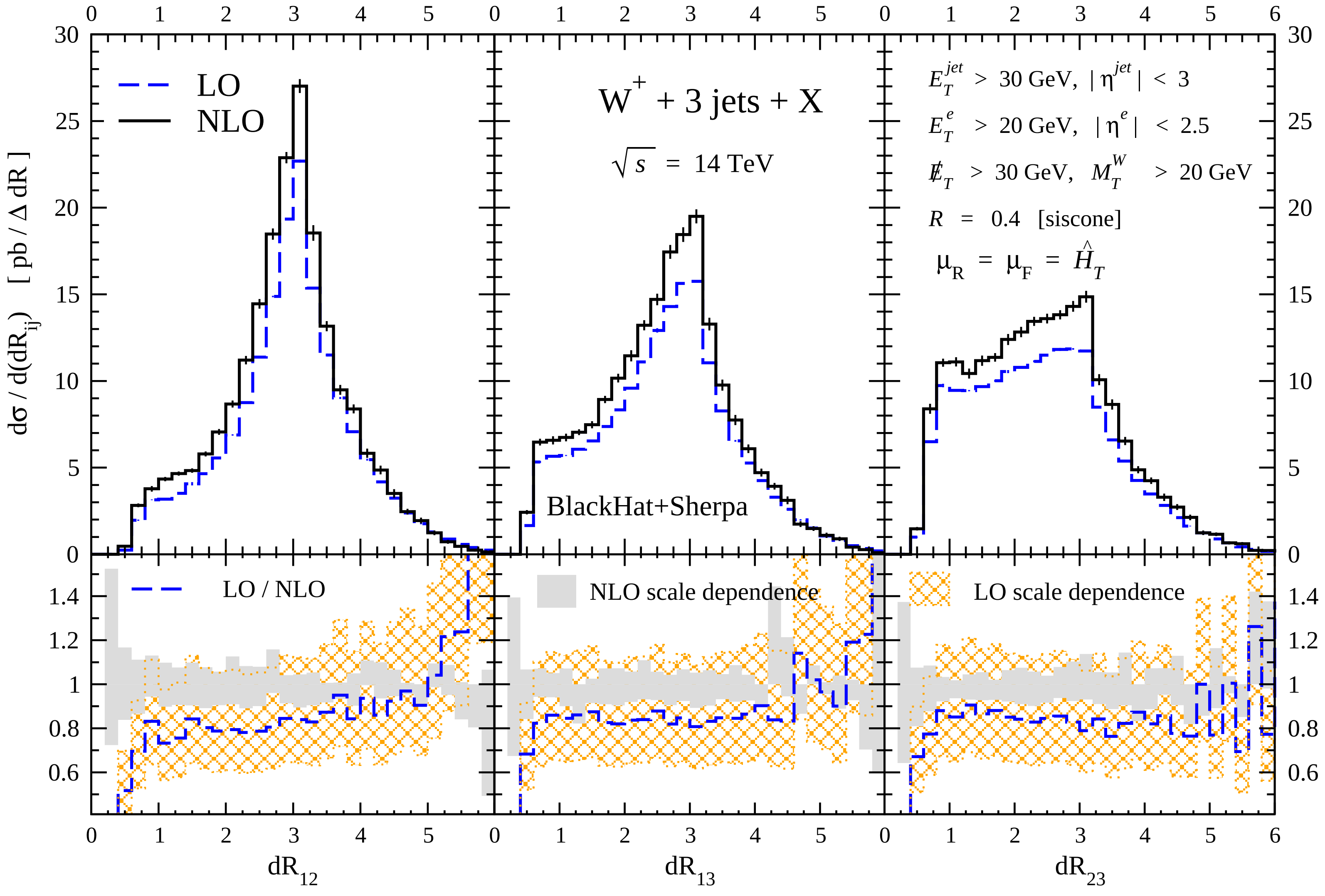
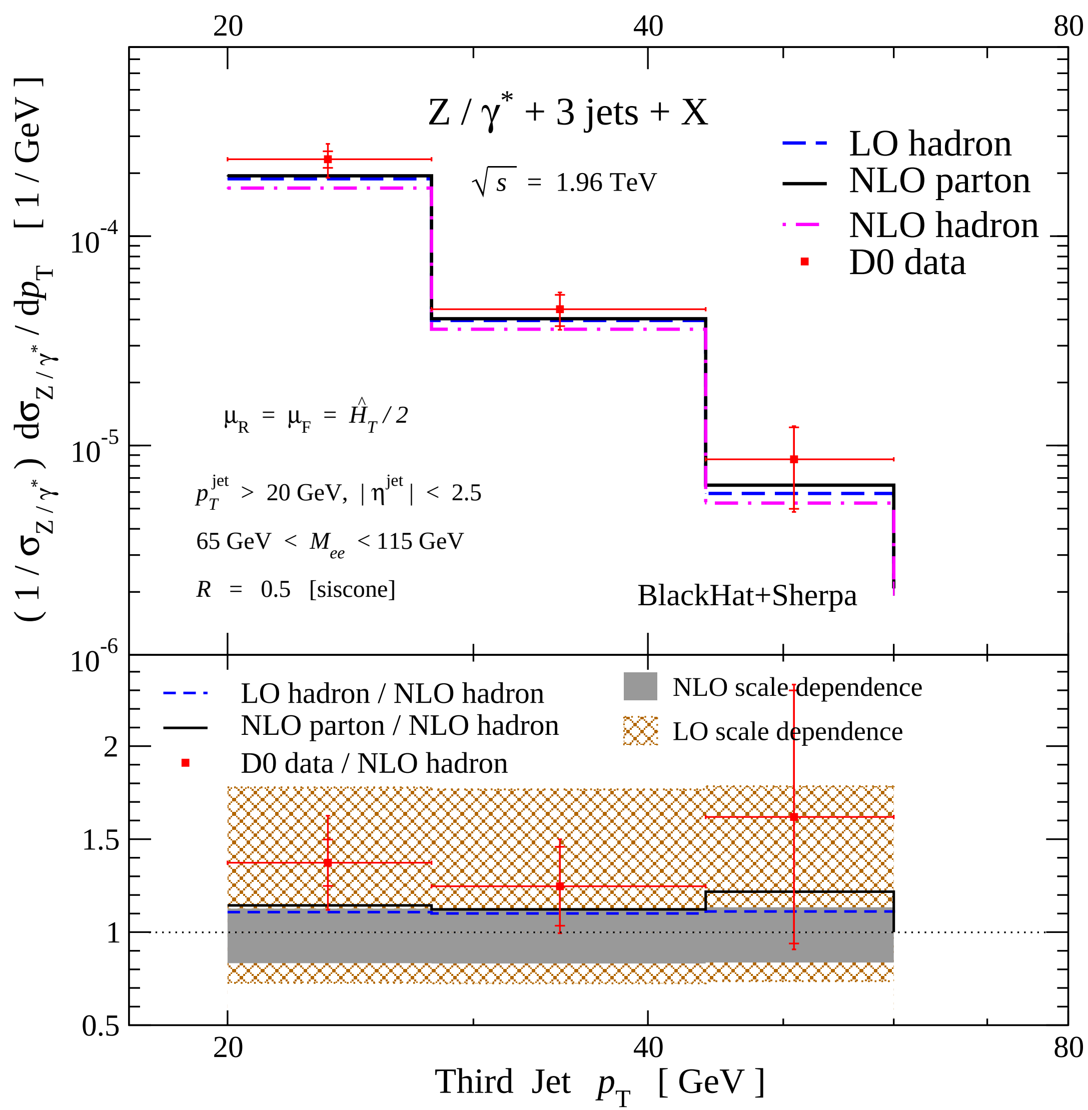



BlackHat + Sherpa

# $Z$ + 3 Jets at the Tevatron



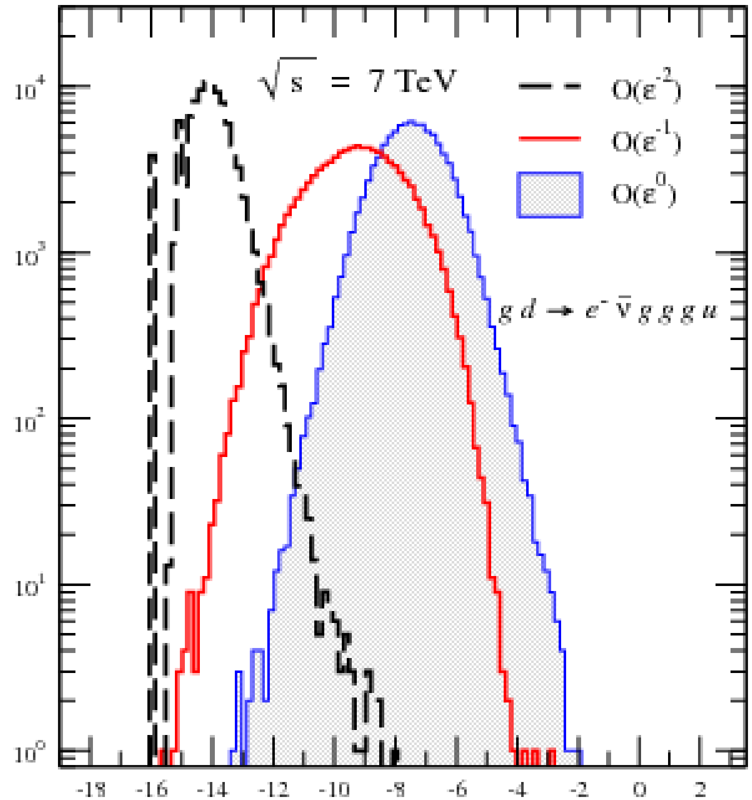



**BlackHat + Sherpa**

# Towards $W$ + 4 Jets

## Numerical stability, virtual

$$\log\left(\frac{d\sigma_V^{BH} - d\sigma_V^{\text{target}}}{d\sigma_V^{\text{target}}}\right)$$




BlackHat preliminary

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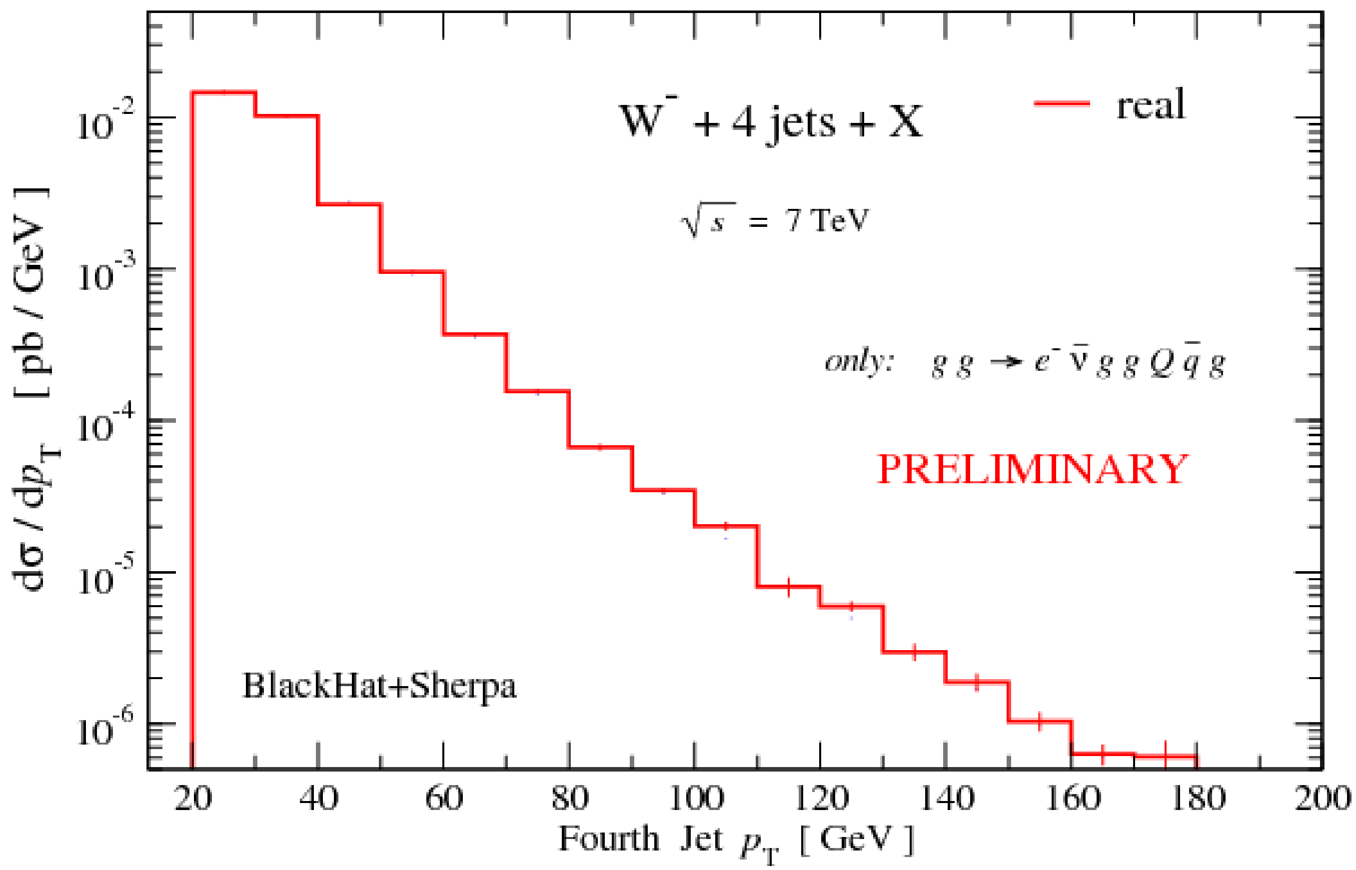
**Summary and Outlook**

## Real emission - $W + 5$ jets with trees from BlackHat




BlackHat + Sherpa preliminary

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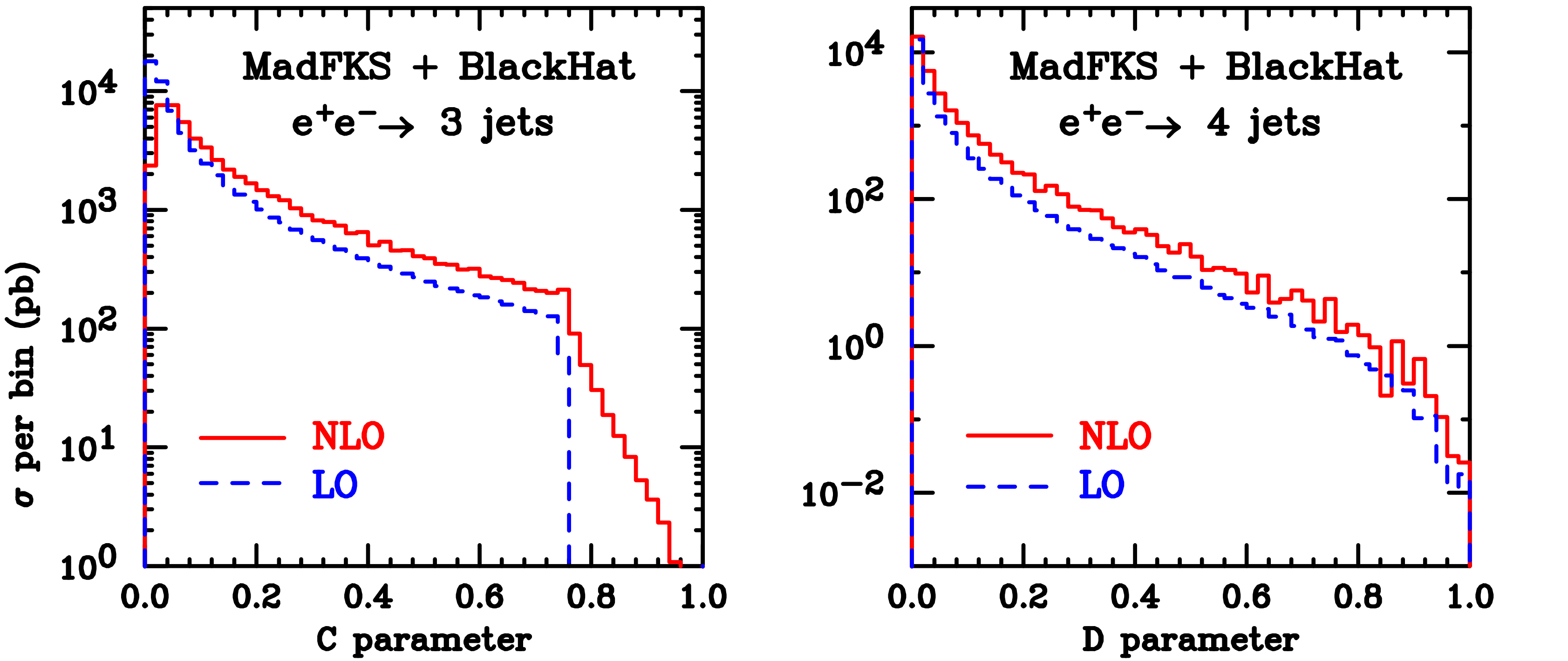


Frederix, Maitre

# Summary and Outlook



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# Summary and Outlook



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# It’s 2010!



# Backup Slides

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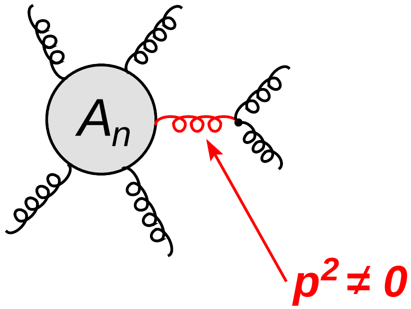
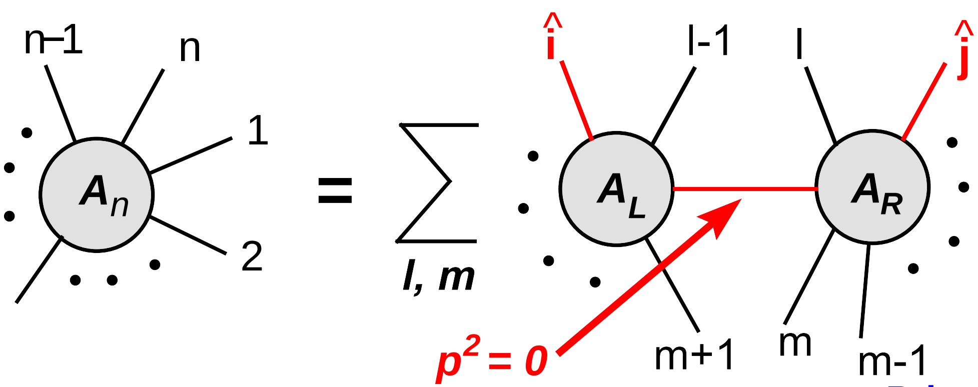
# On-Shell Recursion Relations at Tree Level




**Complex continue (shift) spinors and momenta:**

$$p_i \longrightarrow p_i(z) \qquad p_j \longrightarrow p_j(z)$$
$$p_i + p_j \longrightarrow p_i + p_j$$

**Momentum conservation is maintained, momenta on-shell ($p_i(z)^2 = p_j(z)^2 = 0$).**




Britto, Cachazo, Feng

# Proof at Tree-Level



**Propagators and thus amplitudes are now functions of the complex parameter:**

$$1/P^2_{l\ldots j\ldots m} \quad \rightarrow \quad 1/P^2_{l\ldots j\ldots m}(z)$$

$$A(z) = \sum_{l,m} \sum_{h} A_L^h(z) \frac{1}{P^2_{l\ldots j\ldots m}(z)} A_R^{-h}(z)$$

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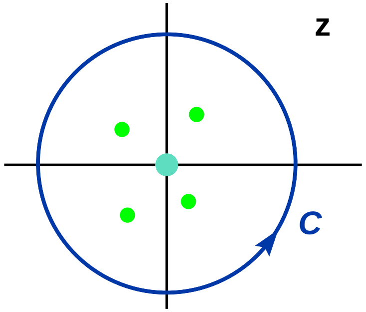
**If** $A(z\rightarrow\infty)\rightarrow 0$ **- Cauchy's theorem**




$$\frac{1}{2\pi i}\oint_C \frac{dz}{z}A(z) = 0$$

$$A(0) = -\sum_{\text{poles }\alpha}\operatorname*{Res}_{z=z_\alpha}\frac{A(z)}{z}$$

$$= \sum_{\text{poles }\alpha}\sum_{h} A_L^h(z_\alpha)\frac{1}{P^2_{l\ldots j\ldots m}}A_R^{-h}(z_\alpha)$$

Britto, Cachazo, Feng, Witten

# From 14 TeV to 10 TeV CM Energy



## Reduction in luminosity approx. 1/2