On-Shell Methods for Collider Physics

Carola F. Berger

*CTP, MIT*

Amplitudes 2010, May 5th, 2010
BlackHat:

CFB, Zvi Bern, Lance Dixon, Fernando Febres Cordero, Darren Forde, Harald Ita, David Kosower, Daniel Maitre


Sherpa liaison (real emissions):

Tanju Gleisberg

It’s 2010!
It’s 2010!

Note the black hats
Outline

- Introduction - Precision Calculations
- On-Shell Methods
  - Generalized unitarity
  - Rational Terms - recursion and D-dimensional unitarity
- Black Magic? BlackHat!
- Summary and Outlook
This talk is about the computation of the hard scattering $\hat{\sigma}$, which is perturbatively calculable for infrared-safe observables.
Why NLO? Example: Single Top

- **“W+2 jet” topology not very distinct**
  - Signal/Background W+2jet+btag ~ 1:17
  - Counting experiment impossible

- **Many sources of background**
  - Only a few directly estimated from (~NLO) MC
  - Data driven and “hybrid” (data+MC)
    - large syst. uncertainties!

- **No “golden” variable**
  - Signal distributions and background distributions look similar

---

From R. Wallny’s (CDF) Fermilab Wine and Cheese 2009
Why NLO? Example: Single Top - Shape!!

\[ \frac{d\sigma}{d\eta} \text{ [pb/}\Delta\eta] \]

-2.4 -1.6 -0.8 0 0.8 1.6 2.4

\( \frac{d\sigma}{d\eta} \) vs \( \eta \) for W + 2 jets

LO / NLO

BlackHat + Sherpa

CFB et al (BlackHat + SHERPA)
Why NLO? Example: Single Top - Shape!!

Leading jet $p_T$ distribution of $Wb\bar{b}$ events
LO dashed, NLO solid

```
pp → $e^+\nu_e b\bar{b} + X$

pp → $e^-\bar{\nu}_e b\bar{b} + X$

$W^+jj/W^+b\bar{b}$

$W^-jj/W^-b\bar{b}$
```

---

Campbell, Ellis, Rainwater
Why NLO?

From the Tevatron to the LHC...
Why NLO?

From the Tevatron to the LHC...
### The (In)Famous Wishlist

<table>
<thead>
<tr>
<th>process wanted at NLO ($V \in {Z, W, \gamma}$)</th>
<th>background to</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $pp \rightarrow VV + \text{ jet}$</td>
<td>$t\bar{t}H$, new physics</td>
</tr>
<tr>
<td>2. $pp \rightarrow H + 2 \text{ jets}$</td>
<td>$H$ production by</td>
</tr>
<tr>
<td>3. $pp \rightarrow t\bar{t}b\bar{b}$</td>
<td>vector boson fusion (VBF)</td>
</tr>
<tr>
<td>4. $pp \rightarrow t\bar{t} + 2 \text{ jets}$</td>
<td>$t\bar{t}H$</td>
</tr>
<tr>
<td>5. $pp \rightarrow VVb\bar{b}$</td>
<td>$t\bar{t}H$</td>
</tr>
<tr>
<td>6. $pp \rightarrow VV + 2 \text{ jets}$</td>
<td>VBF $\rightarrow H \rightarrow VV$, $t\bar{t}H$, new physics</td>
</tr>
<tr>
<td>7. $pp \rightarrow V + 3 \text{ jets}$</td>
<td>VBF $\rightarrow H \rightarrow VV$</td>
</tr>
<tr>
<td>8. $pp \rightarrow VVV$</td>
<td>new physics</td>
</tr>
<tr>
<td></td>
<td>SUSY trilepton</td>
</tr>
</tbody>
</table>
### The (In)Famous Wishlist

<table>
<thead>
<tr>
<th>process wanted at NLO ( (V \in {Z, W, \gamma}) )</th>
<th>background to</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( pp \rightarrow VV + \text{jet} )</td>
<td>( t\bar{t}H ), new physics</td>
</tr>
<tr>
<td>2. ( pp \rightarrow H + 2 \text{jets} )</td>
<td>( H ) production by vector boson fusion (VBF)</td>
</tr>
<tr>
<td></td>
<td>( gg: ) Campbell, Ellis, Zanderighi</td>
</tr>
<tr>
<td>3. ( pp \rightarrow t\bar{t}b\bar{b} )</td>
<td>( t\bar{t}H )</td>
</tr>
<tr>
<td>4. ( pp \rightarrow t\bar{t} + 2 \text{jets} )</td>
<td>( t\bar{t}H )</td>
</tr>
<tr>
<td>5. ( pp \rightarrow VVb\bar{b} )</td>
<td>VBF ( \rightarrow H \rightarrow VV ), ( t\bar{t}H ), new physics</td>
</tr>
<tr>
<td>6. ( pp \rightarrow VV + 2 \text{jets} )</td>
<td>( VBF \rightarrow VV )</td>
</tr>
<tr>
<td>7. ( pp \rightarrow V + 3 \text{jets} )</td>
<td>part ( VBF: ) Bozzi, Jäger, Oleari, Zeppenfeld</td>
</tr>
<tr>
<td>8. ( pp \rightarrow VVV )</td>
<td>new physics</td>
</tr>
<tr>
<td>9. ( pp \rightarrow b\bar{b}b\bar{b} )</td>
<td>SUSY trilepton</td>
</tr>
</tbody>
</table>

\( ZZZ: \) Lazopoulos, Melnikov, Petriello

**Summary and Outlook**

- partially completed, via standard methods
## The (In)Famous Wishlist

<table>
<thead>
<tr>
<th>process wanted at NLO</th>
<th>background to</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $pp \rightarrow VV + \text{jet}$</td>
<td>$t\bar{t}H$, new physics</td>
</tr>
<tr>
<td>2. $pp \rightarrow H + 2 \text{jets}$</td>
<td>$H$ in VBF</td>
</tr>
<tr>
<td>3. $pp \rightarrow t\bar{t}b\bar{b}$</td>
<td>$t\bar{t}H$</td>
</tr>
<tr>
<td>4. $pp \rightarrow t\bar{t} + 2 \text{jets}$</td>
<td>$t\bar{t}H$</td>
</tr>
<tr>
<td>5. $pp \rightarrow VVb\bar{b}$</td>
<td>VBF $\rightarrow H \rightarrow VV, t\bar{t}H$, new physics</td>
</tr>
<tr>
<td>6. $pp \rightarrow VV + 2 \text{jets}$</td>
<td>VBF: Bozzi, Jäger, Oleari, Zeppenfeld</td>
</tr>
<tr>
<td>7. $pp \rightarrow V + 3 \text{jets}$</td>
<td>new physics</td>
</tr>
<tr>
<td>8. $pp \rightarrow VVV$</td>
<td>SUSY trilepton</td>
</tr>
<tr>
<td>9. $pp \rightarrow b\bar{b}b\bar{b}$</td>
<td>Higgs, new physics</td>
</tr>
</tbody>
</table>

- Dittmaier, Kallweit, Uwer; Campbell, Ellis, Zanderighi
- Campbell, Ellis, Zanderighi; Ciccolini, Denner Dittmaier
- Bredenstein, Denner Dittmaier, Pozzorini; Bevilacqua, Czakon, Papadopoulos, Pittau, Worek
- Bevilacqua, Czakon, Papadopoulos, Worek
- CFB, Bern, Dixon, Febres Cordero, Forde, Gleisberg, Ita, Kosower, Maitre; Ellis, Melnikov, Zanderighi
- Lazopoulos, Melnikov, Petriello; Hankele, Zeppenfeld; Binoth, Ossola, Papadopoulos, Pittau
- GOLEM
Why Not (Yet) NLO?

One-loop 6-gluon Feynman diagrams:
Why Not (Yet) NLO?

Result for 1 helicity amplitude (rational part only):

Xiao, Yang, Zhu
A Better Way?

Introduction
- Hadron Collisions
- Why NLO?
  Example: Single Top
- The LHC Wishlist
- Why Not (Yet) NLO?
- A Better Way?
- NLO Corrections to LHC Processes

On-Shell Methods
- Black Magic?
- BlackHat!

Summary and Outlook
A Better Way?

Result for the same helicity amplitude (rational part only):

\[
R_{i,j} = \begin{pmatrix}
R_{1,1} & R_{1,2} \\
R_{2,1} & R_{2,2}
\end{pmatrix}
\]

\[
\frac{1}{2} \left( \left[ \frac{\pi^4}{2} \right] + \left[ \frac{\pi^4}{2} \right] \right)
\]

\[
\sum_{i,j} \left( \begin{array}{c}
R_{i,j} \\
R_{j,i}
\end{array} \right)
\]

\[
\text{CFB, Bern, Dixon, Forde, Kosower}
\]
NLO Corrections to LHC Processes

- Relevant processes all $2 \rightarrow n \geq 3$ as listed in the experimenters’ (in)famous Les Houches wishlist
NLO Corrections to LHC Processes

- Relevant processes all $2 \rightarrow n \geq 3$ as listed in the experimenters’ (in)famous Les Houches wishlist
- Real-virtual cancellations a solved problem, automated
NLO Corrections to LHC Processes

- Relevant processes all $2 \rightarrow n \geq 3$ as listed in the experimenters’ (in)famous Les Houches wishlist
- Real-virtual cancellations a solved problem, automated
- **Bottleneck:** 1-loop virtual amplitudes
  It took 12 years to go from 5-gluon 1-loop amplitudes to 6 gluons!
NLO Corrections to LHC Processes

- Relevant processes all $2 \rightarrow n \geq 3$ as listed in the experimenters’ (in)famous Les Houches wishlist
- Real-virtual cancellations a solved problem, automated
- **Bottleneck**: 1-loop virtual amplitudes
  It took 12 years to go from 5-gluon 1-loop amplitudes to 6 gluons!
- **New methods** based on (generalized) unitarity and recursion $\Rightarrow$ new codes:
  - **BlackHat**
    CFB, Bern, Dixon, Febres Cordero, Forde, Ita, Kosower, Maitre
  - **Rocket** (D-dim unitarity)
    Ellis, Giele, Kunszt, Melnikov, Zanderighi
  - **CutTools/OneLOop** (D-dim unitarity at integrand level)
    van Hameren, Ossola, Papadopoulos, Pittau
NLO Corrections to LHC Processes

- Relevant processes all $2 \rightarrow n \geq 3$ as listed in the experimenters’ (in)famous Les Houches wishlist
- Real-virtual cancellations a solved problem, automated
- **Bottleneck:** 1-loop virtual amplitudes
  It took 12 years to go from 5-gluon 1-loop amplitudes to 6 gluons!
- **New methods** based on (generalized) unitarity and recursion $\Rightarrow$ new codes:
  - **BlackHat**
    - CFB, Bern, Dixon, Febres Cordero, Forde, Ita, Kosower, Maitre
  - **Rocket (D-dim unitarity)**
    - Ellis, Giele, Kunszt, Melnikov, Zanderighi
  - **CutTools/OneLOop (D-dim unitarity at integrand level)**
    - van Hameren, Ossola, Papadopoulos, Pittau
On-Shell Methods

Introduction

On-Shell Methods

- One-Loop Decomposition
- Generalized Unitarity
- Disentangling Coefficients
- Rational Terms from Recursion
- Recursion at Loop Level
- D-dim Unitarity
- BlackHat

Black Magic?
BlackHat!

Summary and Outlook
Any $n$-leg (massless) one-loop amplitude expressible in terms of scalar box, triangle and bubble integrals:

$$A = c_4 I_4 + c_3 I_3 + c_2 I_2 + \text{ rational}$$

With massive partons there are additionally $I_1$ (tadpoles)

We know the integrals, the task is to determine the coefficients

Bern, Dixon, Dunbar, Kosower
Generalized Unitarity

\[ c_4 I_4 = c_4 \int \frac{1}{d^4l} \frac{1}{l^2(l - K_1)^2(l - K_2)^2(l - K_3)^2} \]

\[ \frac{1}{P^2 + i\epsilon} = \frac{1}{P^2} + i\delta^+(P^2) \]

Box integrals have unique leading singularity \( \Rightarrow \) generalized unitarity

\[ c_4 \Delta_{LS} I_4 = \int \frac{1}{d^4l} \delta^+(l^2) \delta^+((l - K_1)^2) \times \delta^+((l - K_2)^2) \delta^+((l - K_3)^2) \times A^{\text{tree}}_1(l) \times A^{\text{tree}}_2(l) \times A^{\text{tree}}_3(l) \times A^{\text{tree}}_4(l) \]

\[ c_4 = A^{\text{tree}}_1(l_{\text{sol}}) \times A^{\text{tree}}_2(l_{\text{sol}}) \times A^{\text{tree}}_3(l_{\text{sol}}) \times A^{\text{tree}}_4(l_{\text{sol}}) \]

Tree graphs on shell

Trees “recycled” into loops

Britto, Cachazo, Feng
Generalized Unitarity contd.

Triangle coefficients from triple cuts, bubble coefficients from double cuts.

\[ \begin{align*}
\text{Triangle coefficients:} & \quad \begin{array}{c}
\includegraphics[width=0.3\textwidth]{triangle_coefficients}
\end{array} \\
\text{Bubble coefficients:} & \quad \begin{array}{c}
\includegraphics[width=0.3\textwidth]{bubble_coefficients}
\end{array}
\end{align*} \]

But life’s not so simple – “leakage” from higher-point integrals into lower point ones because integrals are not fully localized any more.

However, the singularity structures are unique – need procedure to disentangle coefficients:

Clever parametrization of integral – read off coefficients directly

Forde; Ossola, Papadopoulos, Pittau; Kilgore
Disentangling Coefficients

Parametrization of loop momenta (schematically):

\[ l_n^\mu = \alpha_1 K_1^\mu + \alpha_2 K_2^\mu + \alpha_3 t K_3(K_1, K_2)^\mu + \frac{\alpha_4}{t} K_4(K_1, K_2)^\mu \]

\[
\begin{align*}
C_3 &= \sum_{j=-3}^{3} c_j t^j + \sum_i b_i \xi_i (t - t_i) \\
l_i^2(t) &\sim \xi_i (t - t_i).
\end{align*}
\]

Boxes have extra poles in \( t \) from propagators that go on-shell. But we know the boxes, so subtract them off.
Triangle contributions after subtraction of boxes:

\[ T_3 = \sum_{j=-3}^{3} c_j t^j \]

\( c_0 \) is the triangle coefficient, extract via discrete Fourier transform

\[ c_0 = \frac{1}{7} \sum_{j=0}^{6} T_3 \left( t_0 e^{2\pi i j / 7} \right) \]
Rational Terms from Recursion

\[ \mathcal{A} = \sum_i c_i I_i + \text{rational} \]

\[ R = \sum_{\text{configs}} A_L \frac{1}{P^2_{l...m}} A_R \]

Not as simple as the analogous (BCFW) tree level recursion,

Alternative approach (also in BlackHat):

\( D \)-dimensional unitarity

CFB, Bern, Dixon, Forde, Kosower
Recursion at Loop Level

Complex continue amplitude

\[ A(z) = C(z) + R(z) \]
\[ A(0) = C(0) - \sum_{\text{poles } \alpha} \text{Res}_{z=z_{\alpha}} \frac{R(z)}{z} \]
\[ = C(0) + \sum_{\text{configs}} A_L \frac{1}{P_{l...m}^2} A_R \]

Loops “recycled” into loops (ignoring several subtleties)

CFB, Bern, Dixon, Forde, Kosower
Rational Terms - D-dim Unitarity

Unitarity in $D = 4 - 2\varepsilon$:
Split up into 4-D piece and $(-2\varepsilon)$-dim. piece ($\sim$ small “mass”)

\[ l^2_D = l^2_4 + l^2_{[-2\varepsilon]} = l_4^2 + \mu^2 \]

\[ \int \frac{d^D l}{(2\pi)^D} = \int \frac{d^4 l_4}{(2\pi)^4} \int \frac{d^{-\varepsilon}(\mu^2)}{(2\pi)^{-2\varepsilon}} \]

Extract rational part $R$ by keeping track of $\mu$-dependence in generalized unitarity cuts:

\[ A = c_4^{[0]} I_4^D [1] + c_4^{[2]} I_4^D [\mu^2] + c_4^{[4]} I_4^D [\mu^4] + c_3^{[0]} I_3^D [1] + \ldots \]

\[ I_n^D [\mu^{2r}] = \frac{1}{2^r} I_n^{D+2r} [1] \prod_{k=0}^{r-1} (D - 4 + k) \]
Rational Terms - D-dim Unitarity contd.

\[ \mathcal{A} = c_4^{[0]} I_4^D [1] + c_4^{[2]} I_4^D [\mu^2] + c_4^{[4]} I_4^D [\mu^4] + c_3^{[0]} I_3^D [1] + \ldots \]

\[ \mathcal{A} = c_4^{[0]} I_4^D + \frac{D - 4}{2} c_4^{[2]} I_4^{D+2} \]

\[ + \frac{(D - 4)(D - 2)}{4} c_4^{[4]} I_4^{D+4} + c_3^{[0]} I_3^D + \ldots \]

\[ = c_4^{[0]} I_4^{4-2\varepsilon} + c_3^{[0]} I_3^{4-2\varepsilon} + c_2^{[0]} I_2^{4-2\varepsilon} + R \]

\[ R = c_4^{[4]} I_4^{4-2\varepsilon} [\mu^4] \bigg|_{\varepsilon=0} + c_3^{[2]} I_3^{4-2\varepsilon} [\mu^4] \bigg|_{\varepsilon=0} + \ldots \]

Badger, Forde. See also Ossola, Papadopoulos, Pittau (CutTools); Ellis, Giele, Kunszt, Melnikov, Zanderighi (Rocket).
\[ A = \sum_i c_i I_i + \text{rational} \]

- **Cut parts from 4-D unitarity**
\[ A = \sum_i c_i I_i + \text{rational} \]

- Cut parts from 4-D unitarity
- Rational parts from loop recursion
\[ A = \sum_i c_i I_i + \text{rational} \]

- Cut parts from 4-D unitarity
- Rational parts from loop recursion
- OR rational parts from D-dim unitarity
  \[ \Rightarrow \text{4-D unitarity with small “mass”} \]
\[ A = \sum_i c_i I_i + \text{rational} \]

- Cut parts from 4-D unitarity
- Rational parts from loop recursion
- OR rational parts from D-dim unitarity
  \[ \Rightarrow \text{4-D unitarity with small “mass”} \]
- Basic ingredients: tree amplitudes, low-point 1-loop amplitudes
\[ A = \sum_i c_i I_i + \text{rational} \]

- Cut parts from 4-D unitarity
- Rational parts from loop recursion
- OR rational parts from D-dim unitarity  
  \[ \Rightarrow 4\text{-D unitarity with small “mass”} \]
- Basic ingredients: tree amplitudes, low-point 1-loop amplitudes
- NO integrals or PV reductions are performed  
  \[ \Rightarrow \text{Numerically very stable, excellent scaling with number of external legs (number of Feynman graphs grows factorially)} \]
\[ A = \sum_{i} c_i I_i + \text{rational} \]

- Cut parts from 4-D unitarity
- Rational parts from loop recursion
- OR rational parts from D-dim unitarity → 4-D unitarity with small “mass”
- Basic ingredients: tree amplitudes, low-point 1-loop amplitudes
- NO integrals or PV reductions are performed → Numerically very stable, excellent scaling with number of external legs (number of Feynman graphs grows factorially)

⇒ summary in review: CFB, Forde, arXiv:0912.3534 (ARNPS)
Black Magic? BlackHat! – Results

On-Shell Methods

Black Magic? BlackHat!

- Tevatron
- $W$s and $Z$s
- $W + 3$ Jets at the LHC
- $Z + 3$ Jets at the Tevatron
- Towards $W + 4$ Jets
- More to Come!

Summary and Outlook
$W + 1, 2, 3$ jets at the Tevatron

Introduction

On-Shell Methods

Black Magic? BlackHat!

Tevatron

$W$s and $Z$s

$W + 3$ Jets at the LHC

$Z + 3$ Jets at the Tevatron

Towards $W + 4$ Jets

More to Come!

Summary and Outlook

No MCFM for $W + 3$ jets

CDF 2007
$W + 3$ jets at the Tevatron – BlackHat

![Graph showing $W + 3$ jets at the Tevatron](Image)

- **BlackHat and Sherpa**
- **It’s 2010!**
- **Outline**

**Introduction**

**On-Shell Methods**

- **Black Magic? BlackHat!**
  - **Tevatron**
  - $W$s and $Z$s
  - $W + 3$ Jets at the LHC
  - $Z + 3$ Jets at the Tevatron
  - Towards $W + 4$ Jets
  - More to Come!

**Summary and Outlook**

---

**Carola F. Berger**

Amplitudes 2010, May 5th, 2010

On-Shell Methods for Collider Physics - 28/36
**Ws and Zs**

- **W(→ lν) + jets and Z(→ νν) + jets** significant (irreducible) backgrounds in searches for new physics
- **Z → νν**-background can be calibrated (“data-driven”) from **Z → l⁺l⁻**, but especially in initial LHC running needs to be supplemented by theory input (Monte Carlos,...) because of very low statistics
- **W → lν** has larger cross section, but is less clean because of intrinsic missing energy
- Underlying QCD dynamics is the same for both ⇒ can use **Z/W** to calibrate **W/Z**
- **ATLAS and CMS** estimate uncertainty up to 30-50% due to (LO) Monte Carlo input
$W + 3$ Jets at the LHC

Introduction

On-Shell Methods

Black Magic?
BlackHat!

Tevatron
$W$ s and $Z$ s
$W + 3$ Jets at the LHC
$Z + 3$ Jets at the Tevatron
Towards $W + 4$ Jets
More to Come!

Summary and Outlook
**$W + 3$ Jets at the LHC**

**Outline**
- Introduction
- On-Shell Methods
  - Black Magic? BlackHat!
  - Tevatron
  - $W$'s and $Z$'s
  - $W + 3$ Jets at the LHC
  - $Z + 3$ Jets at the Tevatron
  - Towards $W + 4$ Jets
  - More to Come!
- Summary and Outlook

**Introduction**

**On-Shell Methods**
- BlackHat and Sherpa
- It's 2010!
- Outline

**Black Magic? BlackHat!**
- Tevatron
- $W$'s and $Z$'s
- $W + 3$ Jets at the LHC
- $Z + 3$ Jets at the Tevatron
- Towards $W + 4$ Jets
- More to Come!

**Summary and Outlook**

---

- $W^+ + 3$ jets + X
- $\sqrt{s} = 14$ TeV
- $E_T^{jet} > 30$ GeV, $|\eta^{jet}| < 3$
- $E_T^{e} > 20$ GeV, $|\eta^{e}| < 2.5$
- $E_T^{/}\gamma > 30$ GeV, $M^W_T > 20$ GeV
- $R = 0.4$ [siscone]
- $\mu_R = \mu_F = H_T$

---

**BlackHat + Sherpa**
$Z + 3$ Jets at the Tevatron

\[ \frac{1}{\sigma_{Z/\gamma^*}} \frac{d\sigma_{Z/\gamma^*}}{dp_T} [1/\text{GeV}] \]

- $\sqrt{s} = 1.96$ TeV
- $p_T^{\text{jet}} > 20$ GeV, $|\eta^{\text{jet}}| < 2.5$
- $65$ GeV < $M_{ee}$ < $115$ GeV
- $R = 0.5$ [siscone]

$\mu_R = \mu_F = \hat{H}_T/2$

- LO hadron
- NLO parton
- NLO hadron
- D0 data

Summary and Outlook

Carola F. Berger
Amplitudes 2010, May 5th, 2010
Towards $W + 4$ Jets

Numerical stability, virtual

$$\log \left( \frac{d\sigma_{BH}^V - d\sigma_{V}^\text{target}}{d\sigma_{V}^\text{target}} \right)$$
Towards $W + 4$ Jets

Real emission - $W + 5$ jets with trees from BlackHat
More to Come!
More to Come!

MadFKS + BlackHat

- $e^+e^- \rightarrow 3 \text{ jets}$
- $e^+e^- \rightarrow 4 \text{ jets}$

NLO
LO

Frederix, Maitre
NLO corrections are important at the LHC
NLO corrections are important at the LHC

Bottleneck so far: 1-loop amplitudes – cannot just press a button
Summary and Outlook

- NLO corrections are important at the LHC
- Bottleneck so far: 1-loop amplitudes – cannot just press a button
- Unitarity methods cleared bottleneck, automatization well underway
Summary and Outlook

- **NLO corrections are important at the LHC**
- **Bottleneck** so far: 1-loop amplitudes – cannot just press a button
- **Unitarity methods** cleared bottleneck, automatization well underway
- **Results:** First complete calculation of $V+3$ jets with BlackHat, $W+4$ jets underway
NLO corrections are important at the LHC

**Bottleneck** so far: 1-loop amplitudes – cannot just press a button

**Unitarity methods** cleared bottleneck, automatization well underway

**Results**: First complete calculation of $V + 3$ jets with BlackHat, $W + 4$ jets underway

**Public BH release** coming soon to a website near you
NLO corrections are important at the LHC

Bottleneck so far: 1-loop amplitudes – cannot just press a button

Unitarity methods cleared bottleneck, automatization well underway

Results: First complete calculation of $V + 3$ jets with BlackHat, $W + 4$ jets underway

Public BH release coming soon to a website near you

Underlying principle: gauge theory structure much simpler than expansion in terms of Feynman diagrams $\Rightarrow$ reorganization of quantum field theoretic description – see talks at this workshop
Summary and Outlook

- **NLO corrections are important at the LHC**
- **Bottleneck** so far: 1-loop amplitudes – cannot just press a button
- **Unitarity methods** cleared bottleneck, automatization well underway
- **Results**: First complete calculation of $V + 3$ jets with BlackHat, $W + 4$ jets underway
- **Public BH release** coming soon to a website near you
- **Underlying principle**: gauge theory structure much simpler than expansion in terms of Feynman diagrams $\Rightarrow$ reorganization of quantum field theoretic description – see talks at this workshop
It’s 2010!
Black Hat University 2.0

Get Access To A Secret Community Where Scripts Applications, Software & Tactics Are Used By "Underground" Marketers To Silently Make Fortunes On The Internet

Enter Your Name and Email Address Below and You'll be Given FIRST Priority When We RE-LAUNCH BHU 2.0
Complex continue (shift) spinors and momenta:

\[ p_i \rightarrow p_i(z) \quad p_j \rightarrow p_j(z) \]

\[ p_i + p_j \rightarrow p_i + p_j \]

Momentum conservation is maintained, momenta on-shell \((p_i(z)^2 = p_j(z)^2 = 0)\).
Proof at Tree-Level

Propagators and thus amplitudes are now functions of the complex parameter:

\[ \frac{1}{P^2_{l...j...m}} \rightarrow \frac{1}{P^2_{l...j...m}(z)} \]

\[ A(z) = \sum_{l,m} \sum_{h} A_h^L(z) \frac{1}{P^2_{l...j...m}(z)} A_R^h(z) \]
Proof at Tree-Level

Propagators and thus amplitudes are now functions of the complex parameter:

\[
\frac{1}{P_{l...j...m}^2} \rightarrow \frac{1}{P_{l...j...m}^2(z)}
\]

\[
A(z) = \sum_{l,m} \sum_h A^h_L(z) \frac{1}{P_{l...j...m}^2(z)} A^{-h}_R(z)
\]

If \( A(z \rightarrow \infty) \rightarrow 0 \) - Cauchy’s theorem

\[
\frac{1}{2\pi i} \oint_C \frac{dz}{z} A(z) = 0
\]

\[
A(0) = - \sum_{\text{poles } \alpha} \text{Res}_{z=z_\alpha} \frac{A(z)}{z} = \sum_{\text{poles } \alpha} \sum_h A^h_L(z_\alpha) \frac{1}{P_{l...j...m}^2} A^{-h}_R(z_\alpha)
\]

Britto, Cachazo, Feng, Witten
From 14 TeV to 10 TeV CM Energy

Reduction in luminosity approx. 1/2

ratios of parton luminosities at 10 TeV LHC and 14 TeV LHC

\[ \text{luminosity ratio} \]

\[ M_X \text{ (GeV)} \]

\[ \Sigma q\bar{q} \]

\[ gg \]

MSTW2008NLO