

Scattering on the Coulomb branch of $\mathcal{N} = 4$ super Yang-Mills

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based on

- F. Alday, J. H., J. Plefka and T. Schuster, [arXiv:0908.0684](#) [hep-th]
J. H., S. Naculich, H. Schnitzer and M. Spradlin, [arXiv:1001.1358](#) [hep-th]
J. H., S. Naculich, H. Schnitzer and M. Spradlin, [arXiv:1004.5381](#) [hep-th]

Amplitudes 2010, London, May 7th

Scattering on the Coulomb branch of $\mathcal{N} = 4$ SYM

- Infrared divergences; Exponentiation
- Extended dual conformal symmetry
- Integral basis at higher loops
- Regge limit

Planar amplitudes on the Coulomb branch of $\mathcal{N} = 4$ Super Yang-Mills

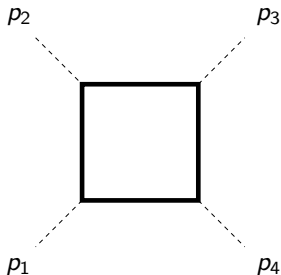
- $U(N + M) \rightarrow U(N) \times U(M)$

[Alday, Maldacena, 2007; Kawai, Suyama, 2007; Schabinger, 2008; Sever, McGreevy, 2008]

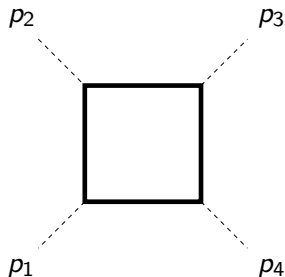
[Alday, J.H., Plefka, Schuster, 2009]

→ leads to massive particles

- scatter massless $U(M)$ particles
- $N \gg M$: only allow loops in N -part of $U(N + M)$
 - renders amplitudes IR finite
- e.g. colour-ordered one-loop amplitude



One-loop example



$$M^{(1)} \propto [\log^2(\dots) + \log(\dots) \log(\dots) + \text{Li}_2(\dots)]$$

in dimensional regularization: $M^{(1)} \propto$ hypergeometric function

One-loop example

Various interesting limits

- Regge limit $s \gg t, m^2$

$$M^{(1)} = \log(s/m^2)\alpha(t/m^2) + O(s^0), \quad \alpha \text{ is Regge trajectory}$$

- large mass limit $m^2 \gg s, t$

[cf. Gorsky and Zhiboedov 2009]

- small mass limit $m^2 \ll s, t$ (“mass regulator”)

$$M^{(1)} = - \left[\frac{1}{2} \log^2 \frac{s}{m^2} + \frac{1}{2} \log^2 \frac{t}{m^2} \right] + \frac{1}{2} \log^2 \left(\frac{s}{t} \right) + \frac{1}{2} \pi^2 + \mathcal{O}(m^2)$$

reminder: in dimensional regularization

$$M^{(1)} = - \left[\frac{1}{2\epsilon^2} \left(\frac{\mu^2}{s} \right)^\epsilon + \frac{1}{2\epsilon^2} \left(\frac{\mu^2}{t} \right)^\epsilon \right] + \frac{1}{2} \log^2 \left(\frac{s}{t} \right) + \frac{2}{3} \pi^2 + \mathcal{O}(\epsilon)$$

Comments:

- geometrical interpretation as volume of tetrahedron in AdS_5

[Mason, Skinner 2010; see also Davydychev and Delbourgo 1998]

- useful in connection with momentum twistor space

[Hodges 2010]

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Exponentiation in Higgs regularization

- universal planar structure

$$\log M_4 = D(s) + D(t) + F_4(s/t) + \mathcal{O}(\epsilon)$$

- reminder: dimensional regularization ($\beta = 0$)

$$D(s) = -1/2 \sum a^\ell \left[\Gamma_{\text{cusp}}^{(\ell)} / (\ell\epsilon)^2 + \mathcal{G}_0^{(\ell)} / (\ell\epsilon) \right] (\mu^2/s)^{\ell\epsilon}$$

- finite part $F_4(s/t)$ is also simple!

[Anastasiou, Bern, Dixon, Kosower 2002; Bern, Dixon, Smirnov, 2003]

$$F_4 = \frac{1}{2} \Gamma_{\text{cusp}}(a) \left[\frac{1}{2} \log^2 \frac{s}{t} + \frac{2}{3} \pi^2 \right] + c(a)$$

$M^{(2)} - \frac{1}{2} (M^{(1)})^2$ interference $1/\epsilon \times \mathcal{O}(\epsilon) = \mathcal{O}(1)$

\Rightarrow in order to compute $\log M$, need $\mathcal{O}(\epsilon)$ terms in M

- analog in Higgs regularization

[Alday, J. H., Plefka, Schuster, 2009; J. H., Naculich, Schnitzer, Spradlin, 2010]

[based on general field theory analysis by Korchemsky, Sterman, ...]

$$D(s) = -\frac{1}{4} \Gamma_{\text{cusp}}(a) \log^2 \frac{s}{m^2} - \tilde{G}_0(a) \log \frac{s}{m^2}$$

F_4 equal up to scheme-dependent constant

we have $m^2 \times \log m^2 \rightarrow 0 \Rightarrow$ can drop all $\mathcal{O}(m^2)$ terms

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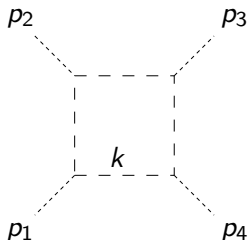
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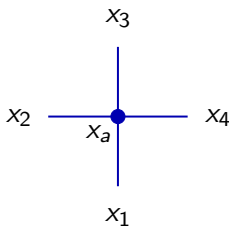
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Dual conformal symmetry (1/2)

- observation: $\mathcal{N} = 4$ SYM loop integrals have a dual conformal symmetry



[Drummond, J.H., Smirnov, Sokatchev, 2006]



- loop integrand has conformal symmetry in **dual space**

$$x_{i+1}^\mu - x_i^\mu = p_i$$

e.g. inversion symmetry $x^\mu \rightarrow x^\mu/x^2$ or special conformal transformations

$$K^\mu = \sum_i \left[2x_i^\mu x_i^\nu \frac{\partial}{\partial x_{i\nu}} - x_i^2 \frac{\partial}{\partial x_{i\mu}} \right]$$

- breaking of symmetry $D = 4 - 2\epsilon$ under control

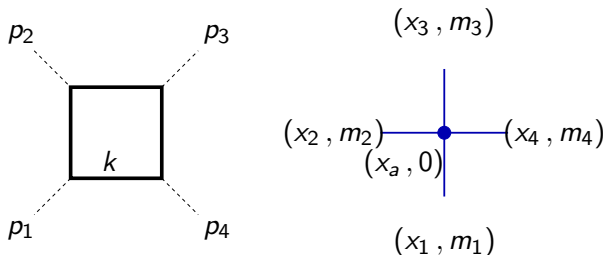
[Drummond, J.H., Korchemsky, Sokatchev, 2007]

Dual conformal symmetry (2/2)

refinement: $U(N + M) \rightarrow U(N) \times U(1)^M$

[Alday, J.H., Plefka, Schuster, 2009]

- on-shell conditions now read $p_i^2 = (m_i - m_{i+1})$
- particles in loop have mass m_i



- **important:** in addition to the dual coordinates x_i , we can vary the masses m_i

$$\hat{K}^\mu = K^\mu + \sum_i \left[2x_i^\mu m_i \frac{\partial}{\partial m_i} - m_i^2 \frac{\partial}{\partial x_{i\mu}} \right]$$

- integral has exact dual conformal symmetry

$$\hat{K}^\mu I = 0$$

- very natural from string theory: m corresponds to radial coordinate of AdS_5

Implications for higher loop integral basis

- basis of loop integrals in $\mathcal{N} = 4$ SYM constrained by dual conformal symmetry?

[Drummond, J.H., Smirnov, Sokatchev, 2006; Bern, Czakon, Dixon, Kosower, Smirnov, 2006; Bern, Carrasco, Johansson, Kosower, 2007]

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- it seems reasonable to speculate that

[J.H., Naculich, Schnitzer, Spradlin, 2010]

$$M_n = 1 + \sum_{\mathcal{I}} a^{L(\mathcal{I})} c(\mathcal{I}) \mathcal{I},$$

where: coupling a , loop order $L(\mathcal{I})$

coefficients $c(\mathcal{I}) \Rightarrow$ compute by (generalized) unitarity

integrals $\mathcal{I} \Rightarrow$ restricted set of extended dual conformal integrals

- absence of triangles at one loop

[Boels; also: Schabinger]

- additional constraints from expected IR structure

[Korchemsky, Sterman,...]

$$M_n = \exp \left[-\frac{1}{8} \Gamma_{\text{cusp}}(a) \sum_i \log^2 \frac{s_i}{m^2} - \frac{1}{2} \tilde{G}_0(a) \sum_i \log \frac{s_i}{m^2} + \mathcal{O}(\log^0 m^2) \right]$$

- insights from analytic structure for generic m^2 , and Regge limit(s)?

- further constraints from the (broken) conventional conformal symmetry?

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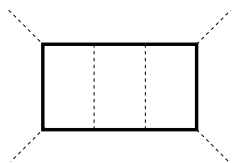
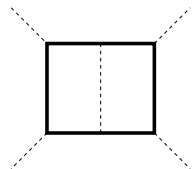
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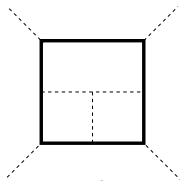
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Extended dual conformal invariance at higher loops

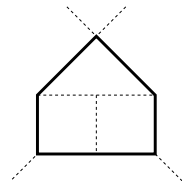
- At 2 loops: Only one integral is allowed by extended dual conformal symmetry:
- At 3 loops: four integrals allowed:



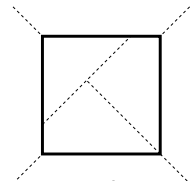
3a



3b



3c



3d

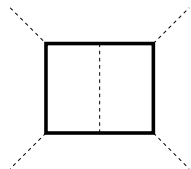
Similarly restricts integral basis at higher loops and legs.

$$M_3 = 1/(-8) [c_{3a}l_{3a} + c_{3b}l_{3b} + c_{3c}l_{3c} + c_{3d}l_{3d} + \{s \leftrightarrow t\}]$$

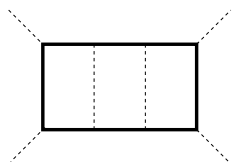
in dimreg: $c_{3a} = 1, c_{3b} = 2$ and $c_{3c} = c_{3d} = 0$

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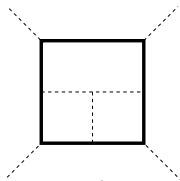
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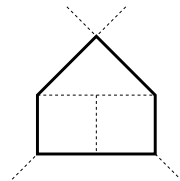
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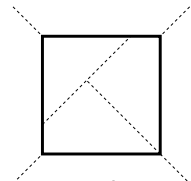
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- analog of BDS in Higgs regularization:

[Alday, J. H., Plefka, Schuster, 2009; J. H., Naculich, Schnitzer, Spradlin, 2010]

$$\begin{aligned} \log M_4 = & -\frac{1}{4}\Gamma_{\text{cusp}}(a) \left[\log^2 \frac{s}{m^2} + \log^2 \frac{t}{m^2} \right] - \tilde{G}_0(a) \left[\log \frac{s}{m^2} + \log \frac{t}{m^2} \right] \\ & + \frac{1}{4}\Gamma_{\text{cusp}}(a) \left[\log^2 \frac{s}{t} + \pi^2 \right] + \tilde{c}(a) + O(m^2) \end{aligned}$$

- verified by computing dual conformal integrals up to $O(m^2)$

- at two loops

[Alday, J. H., Plefka, Schuster, 2009]

- at three loops

[J. H., Naculich, Schnitzer, Spradlin, 2010]

- at four loops (IR divergent terms and Regge limit)

[J. H., Naculich, Schnitzer, Spradlin, 2010]

- method used: MB representation of all integrals, asymptotic expansion, numerical integration (Mathematica packages *MBasymptotics*, *MB*; *CUBA*)

Three-loop exponentiation

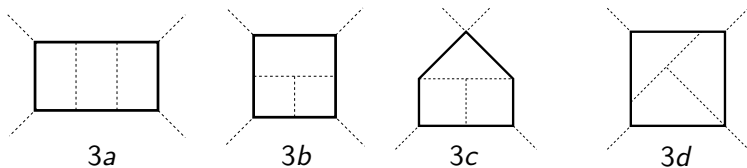
For simplicity, set $s = t$, $\log\left(\frac{m^2}{s}\right) \equiv L$

[J. H., Naculich, Schnitzer, Spradlin, 2010]

Infrared consistency:

$$M^{(3)} = -\frac{1}{6}L^6 + \frac{\pi^2}{12}L^4 + 2\zeta_3 L^3 + \left(-\frac{\pi^4}{30} - \frac{\Gamma_{\text{cusp}}^{(3)}}{4}\right)L^2 + \mathcal{O}(L)$$

On the other hand,



$$M_3 = 1/(-4) [c_{3a}l_{3a} + c_{3b}l_{3b} + c_{3c}l_{3c} + c_{3d}l_{3d}]$$

Compute...

$$l_{3a} = \frac{17}{90}L^6 + \frac{\pi^2}{9}L^4 + \dots, \quad l_{3c} = \mathcal{O}(L^0),$$

$$l_{3b} = \frac{43}{180}L^6 - \frac{\pi^2}{9}L^4 + \dots, \quad l_{3d} = \mathcal{O}(L)$$

Hence $c_{3a} = 1$, $c_{3b} = 2$

Three-loop exponentiation

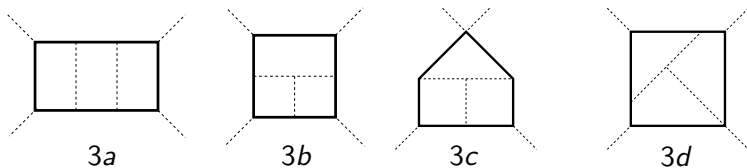
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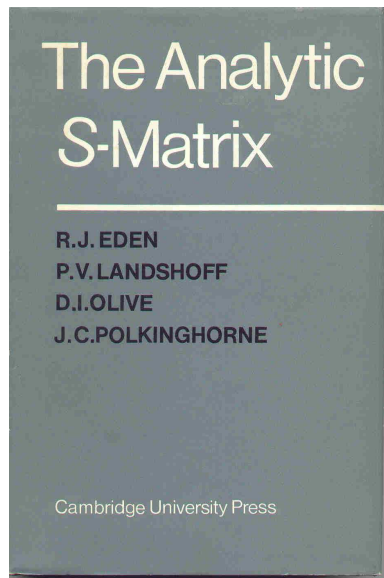
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Regge limits for amplitudes on the Coulomb branch

- take Regge limit $s = (p_1 + p_2)^2 \rightarrow \infty$
expect

[J. H., Naculich, Schnitzer, Spradlin, 2010]

$$\beta(t/m^2) \left(\frac{s}{m^2}\right)^{\alpha(t/m^2)} + \mathcal{O}(m^2)$$

$$\text{trajectory } \alpha(t/m^2) = -\frac{1}{2}\Gamma_{\text{cusp}}(a) \log(t/m^2) - \tilde{\mathcal{G}}_0(a)$$

[Korchensky; ...]

- dual conformal symmetry implies:

$$M(p_i, m_i) = M(u, v), \quad u = \frac{m_1 m_3}{s + (m_1 - m_3)^2}, \quad v = \frac{m_2 m_4}{s + (m_2 - m_4)^2}$$

equal mass case

$$m_i = m$$

$$u = \frac{m^2}{s}, \quad v = \frac{m^2}{t}$$

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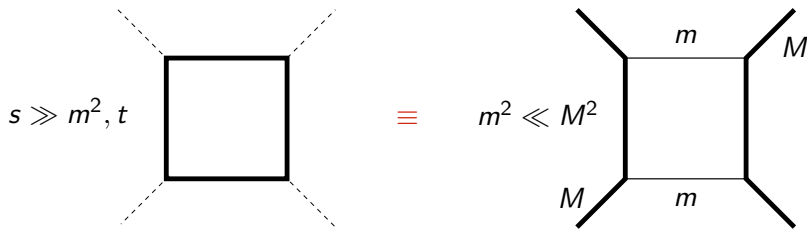
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- determine leading Regge behavior of integrals
- systematics of Regge limit simpler here compared to dimensional regularization

[Eden et al, The analytic S-matrix]

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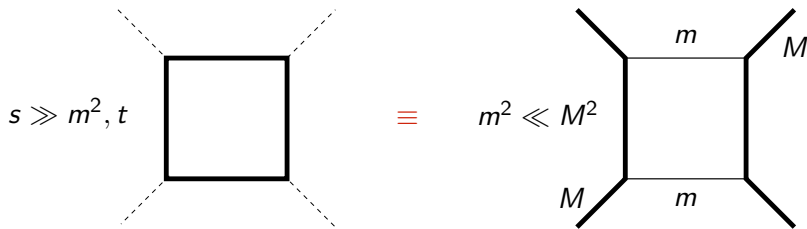
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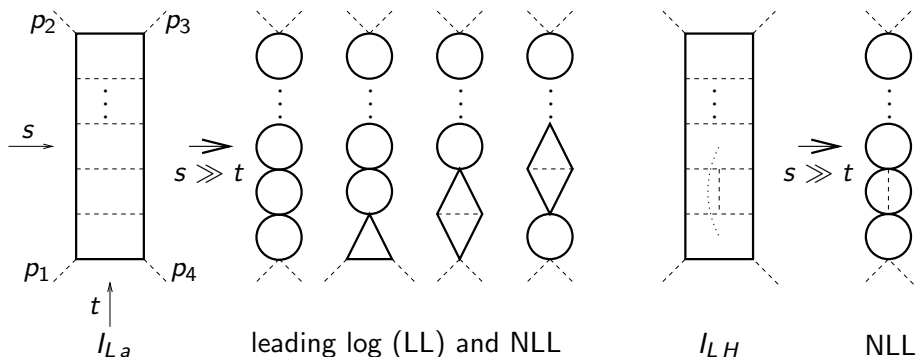
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LL and NLL Regge limit to all loop orders



Regge limit to all loop orders:

- LL : ladder integrals
- NLL and LL : ladders and ladders with one H-shaped insertion

[J. H., Naculich, Schnitzer, Spradlin, 2010 (this monday)]

- in contrast, in dimensional regularization, many different diagrams contribute

- Higgs IR regulator for planar $\mathcal{N} = 4$ SYM
 - makes dual conformal symmetry exact
 - restricts integral basis
 - exponentiation of amplitude easier to compute
 - Regge limit: LL and NLL computed to all loop orders