Holography for a de Sitter-esque Geometry?

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Some words on the static patch

Static patch of de Sitter space:

$$ds^2 = -\left(1 - \frac{r^2}{\ell^2}\right) dt^2 + \left(1 - \frac{r^2}{\ell^2}\right)^{-1} dr^2 + r^2 d\Omega_2^2 \ .$$

Cosmological horizon at $r = \ell$.

Near horizon dynamics = Incompressible non-relativitstic Navier Stokes.

Mysterious Bekenstein-Gibbons-Hawking entropy associated to cosm. horizon $S_{cos} = A/4G_N$. (What is it counting?)

Two parameter deformation (M and a) gives Kerr-de Sitter space.

General rotating black hole solution with positive cosmological constant:

$$ds^2 = -\frac{\Delta_r}{\rho^2}\left(dt - \frac{a}{\Xi}\sin^2\theta d\phi\right)^2 + \frac{\rho^2}{\Delta_r}dr^2 + \frac{\rho^2}{\Delta_\theta}d\theta^2 + \frac{\Delta_\theta}{\rho^2}\sin^2\theta\left(adt - \frac{r^2+a^2}{\Xi}d\phi\right)^2$$

where:

$$\Delta_r = (r^2+a^2)\left(1-\frac{r^2}{\ell^2}\right) - 2Mr, \quad \Xi = 1+\frac{a^2}{\ell^2}\ , \quad \Delta_\theta = 1+\frac{a^2}{\ell^2}\cos^2\theta, \quad \rho^2 = r^2+a^2\cos^2\theta$$

Δ_r has four real roots. Two largest, r_+ and r_c, are black hole and cosmological horizons.

Interesting Regions of Parameter Space

• EXTREMAL BLACK HOLES with vanishing temperature.

• LUKEWARM BLACK HOLES with equal temperature to the cosmological horizon.

• ROTATING NARIAI BLACK HOLES with size equal to the cosmological horizon.

There exist smooth Euclidean instantons corresponding to these geometries [Booth,Mann].

Most Euclideanizations of dS black holes have conical singularities.

Thermodynamics

Mass, angular momentum and cosmological entropy of Kerr-de Sitter:

$$\mathcal{Q}_{\partial_t} = -\frac{M}{\Xi^2}, \quad \mathcal{Q}_{\partial_\phi} = \frac{aM}{\Xi^2}, \quad S_c = \frac{\pi\left(r_c^2 + a^2\right)}{\Xi}$$

Defined at $\mathcal{I}^+$ via regularized Brown-York stress tensor.

First law of thermodynamics in the rotating Nariai limit becomes:

$$dS_c = \beta_L d\mathcal{Q}_{\partial_\phi}, \quad \beta_L = T_L^{-1} \equiv 2\pi k.$$

Near horizon parameters:

$$t' = b\lambda t, \quad r' = \frac{(r - r_+)}{\lambda r_+}, \quad \phi' = \phi - \Omega_{BH} t, \quad \tilde{\tau} \equiv \frac{r_c - r_+}{r_+}.$$

Upon $\lambda \to 0$ with $\tilde{\tau}/\lambda$ fixed:

$$ds^2 = \Gamma(\theta) \left(-r'(\tilde{\tau} - r')dt'^2 + \frac{dr'^2}{r'(\tilde{\tau} - r')} \right) + \gamma(\theta) \left(d\phi + kr'dt' \right)^2 + \alpha(\theta) d\theta^2$$

In global coordinates we find:

$$ds^2 = \Gamma(\theta) \left(-d\tau^2 + \cosh^2 \tau d\psi^2 \right) + \gamma(\theta) \left(d\phi + k \sinh \tau d\psi \right)^2 + \alpha(\theta) d\theta^2$$

with $\psi \sim \psi + 2\pi$. Boundary at $\tau \to +\infty$ is $\mathcal{I}^+_{RN}$.

Geometry of Rotating Nariai

- It is a de Sitter version of the NHEK geometry, i.e. an S^2 fibration over dS_2.
- Constant time slices of the global geometry have an $S^1 \times S^2$ topology.
- There is an $SL(2,\mathbb{R}) \times U(1)$ four-dimensional isometry group.
- When $\tilde{\tau} \neq 0$ both black hole and cosmological horizons are preserved.
- At constant polar angle θ the geometry becomes warped dS_3.

ASG = diffeos obeying certain bc's quotiented by the trivial ones. Trivial = diffeos with vanishing charges at $\mathcal{I}^+_{RN}$.

Boundary conditions at $\mathcal{I}^+_{RN}$ (recall large r is a time coordinate):

$$\begin{aligned} h_{tt} &\sim r^2, \quad h_{\phi\phi} \sim h_{t\phi} \sim 1, \quad h_{t\theta} \sim h_{\phi\theta} \sim h_{\theta\theta} \sim h_{\phi r} \sim 1/r, \\ h_{tr} &\sim h_{\theta r} \sim 1/r^2, \quad h_{rr} \sim 1/r^3. \end{aligned}$$

Diffeos preserving the above boundary conditions are given by:

$$\zeta_n = e^{-in\phi}\left(-\partial_\phi + inr\partial_r\right), \quad \zeta_t = \partial_t.$$

such that $[\zeta_m, \zeta_n] = -i(m-n)\zeta_{m+n}$.

Asymptotic Symmetries II

ASG of rotating Nariai is single copy of Virasoro algebra with REAL, POSITIVE central charge:

$$c_L = \frac{12r_c^2\sqrt{(1-3r_c^2/\ell^2)(1+r_c^2/\ell^2)}}{-1+6r_c^2/\ell^2+3r_c^4\ell^4}.$$

The ASG comes from extending the $U(1)$ isometries.

Central charge vanishes when $r_c^2 = \ell^2/3$, this is the non-rotating Nariai geometry.

Different boundary conditions give rise to a right moving sector.

We propose that rotating Nariai geometry is HOLOGRAPHICALLY DUAL to a two-dimensional Euclidean 'conformal' field theory [D. A.,T. Hartman]:

Indeed, somewhat mysteriously, the cosmological entropy is given by the Cardy formula

$$S_c = \frac{\pi^2}{3} T_L c_L .$$

This is a different class of dualities not continuously connected to dS/CFT or Kerr/CFT.

Beyond $T_L = 0$?

Change in cosmological entropy when we go away from the Nariai limit is POSITIVE.

Leaving the Nariai limit corresponds to adding some right moving energy, thus:

$$S_c = \frac{\pi^2}{3} T_L c_L + \frac{\pi^2}{3} T_R c_R \ .$$

T_R can be read off from quasinormal mode spectrum. Linearly, T_R is temperature of cos. horizon.

Picture is EXACTLY realized for a closely related family of solutions in 3d gravity, warped dS_3 [D. A., S. De Buyl, S. Detournay].

Scalar Waves

We solve [D. A.,T. Anous]:

$$\nabla^2 \Phi = 0 \ ,$$

with ansatz $\Phi(t, r, \phi, \theta) = e^{-i\omega t + im\phi} R(r) Y_{lm}(\theta)$.

Eom's SEPARATE and are solved exactly in terms of HYPERGEOMETRIC FUNCTIONS.

The large r, i.e. late time, behavior goes as:

$$\Phi \sim r^{-h_\pm}, \quad h_\pm = \frac{1}{2} \pm \frac{i}{2}\sqrt{4m^2k^2 + 4j_{lm} - 1}.$$

Thus, conformal weights are COMPLEX.

Quasinormal Modes

Waves with outgoing flux at black hole and cos. horizon, have qnm spectrum:

$$m = -2\pi i T_L(n + h_L) , \quad n = 0, 1, 2, 3, \dots$$
$$\omega = -2\pi i T_R (n + h_R) , \quad n = 0, 1, 2, 3, \dots$$

$T_R = \tilde{\tau}/4\pi$ = temperature of the cosmological horizon in the rotating Nariai geometry and $h_L = h_R = h_\pm$.

These resemble the poles of a thermal retarded Green's function in a CFT_2 (upon identifying the various CFT quantities).

QNM's are poles of correlators at $\mathcal{I}^+_{RN}$ with Dirichlet bc's and flux incoming from a single static patch.