Scaling Solutions and Huge Higg's Homology

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Based on work/discussions/musings with
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many other people

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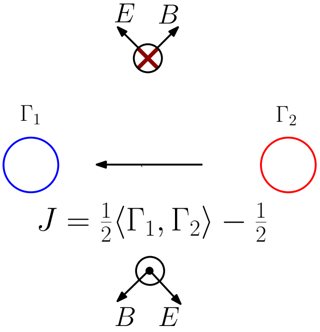
2 centered dyon in N=2 sugra



Number of states: $2J + 1 = \langle\Gamma_1, \Gamma_2\rangle$

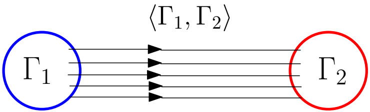
2 centered dyon in N=2 sugra

$$\Delta r = \langle \Gamma_1, \Gamma_2 \rangle \frac{|Z_{1+2}|}{2\mathrm{Im}(Z_1 Z_2)} l_4$$

Note: $l_4 = g_s l_s$

$g_s \to 0$ slightly separated intersecting D-branes

N=4 quiver quantummechanics



Field content

- three scalars x^i (vectormultiplet)
- $\langle\Gamma_1,\Gamma_2\rangle$ scalars ϕ^a (chiral multiplet)

Potential:

$$V(\phi,x) = \left(|\phi|^2+\theta\right)^2 + |x|^2|\phi|^2$$

(Classical) Vacua:

- Coulomb branch: $\phi = 0, \quad x^2 > -\theta$ (quantum corrected = sugra eq's)
- Higg's branch: $x = 0, \quad |\phi|^2 = -\theta$

Higg's branch

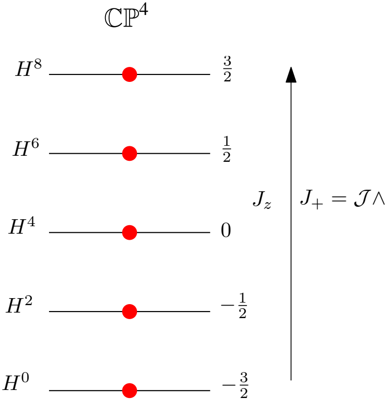
$$|\phi|^2 = -\theta \quad \rightarrow \quad \mathbb{CP}^{\langle\gamma_1,\gamma_2\rangle - 1}$$

Quantumstates $\leftrightarrow$ cohomology elements

$$h^{2i} = 1 \qquad h^{2i+1} = 0$$

$$i = 0, \ldots, \langle\Gamma_1, \Gamma_2\rangle - 1$$

$\langle\Gamma_1, \Gamma_2\rangle$ **states!**



- Matches Coulomb and Sugra description
- Lefschetz SU(2)=angular momentum

Three dyons

$$a = \langle \Gamma_1, \Gamma_2 \rangle \qquad b = \langle \Gamma_2, \Gamma_3 \rangle \qquad c = \langle \Gamma_3, \Gamma_1 \rangle$$

Sugra/Coulomb equations:

$$\begin{aligned} \frac{a}{r_{12}} - \frac{c}{r_{13}} &= \theta_1 \\ -\frac{a}{r_{12}} + \frac{b}{r_{23}} &= \theta_2 \end{aligned}$$

Counting states: $N = (J_{\rm max} - J_{\rm min})(J_{\rm max} + J_{\rm min})$

Scaling solutions: iff $a + b > c$ and cyclic

$$r_{12} = \lambda a, \qquad r_{23} = \lambda b, \qquad r_{31} = \lambda c \qquad \text{is solution for } \lambda \to 0$$

It follows $J_{\rm min} = 0, \qquad N_{\rm scaling} = J_{\rm max}^2$

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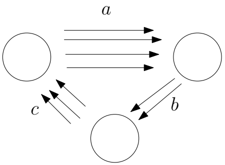
Higg's branch description

Quiver:



Fields

$$\phi_{12}^{i}, \quad i = 1, \ldots, a \qquad \phi_{23}^{j}, \quad j = 1, \ldots, b \qquad \phi_{31}^{k} \quad k = 1, \ldots, c$$

D-term + F-term:

$$\phi_{23} = -\theta_1 \qquad \phi_{31} = -\theta_1 \qquad \sum_{j,k} a_{ijk} \phi_{23}^{j} \phi_{31}^{k} = 0$$

a polynomials in $\mathbb{CP}^{b-1} \times \mathbb{CP}^{c-1}$ $\Rightarrow$ complete intersection manifold

Cohomology growth

Euler characteristic

$$\Omega(a,b,c) = ab - \sum_{i=0}^{a-1}\sum_{j=0}^{b-1}\sum_{k=0}^{c-1}(-1)^{(i+j+k)}\binom{a}{i+1}\binom{b}{j+1}\binom{c}{k+1}\frac{(i+j+k)!}{i!j!k!}$$

- Triangle ineq. violated

$$\text{e.g. } a > b + c \qquad \Rightarrow \qquad \Omega(a,b,c) = a(b-c)$$

- All triangle ineq.'s satisfied:

$$A = -a+b+c > 0, \quad B = a-b+c > 0, \quad C = a+b-c > 0$$

$$\Omega(a,b,c) \sim 2^{a+b+c}\,\frac{a^a b^b c^c}{A^A B^B C^C}\,\frac{\sqrt{abc}\,(ABC)^{3/2}}{(AB+BC+CA)^{7/2}}$$

Exponential growth

Cohomology growth

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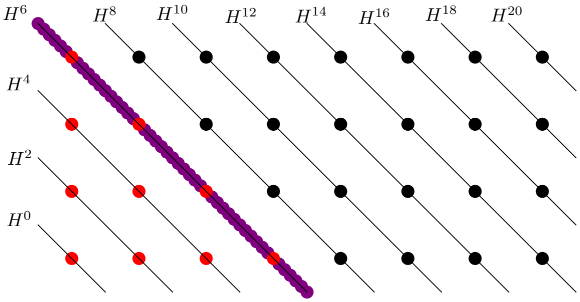
Exponential growth

Precise cohomology structure

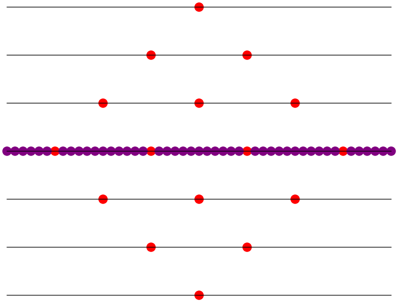
$$X_6 \subset \mathbb{CP}^3 \times \mathbb{CP}^7$$



Lefschetz Hyperplane theorem:

$$H^i(X_a) = H^i(\mathbb{CP}^{b-1} \times \mathbb{CP}^{c-1}) \qquad \forall i < b+c-2-a$$
$$H^i(X_a) \supset H^i(\mathbb{CP}^{b-1} \times \mathbb{CP}^{c-1}) \qquad i = b+c-2-a$$

Higg's vs Coulomb/Sugra



Coulomb states are all there:

$$N_{\text{Coulomb}} = J^2_{\text{max}} = 16$$

But there is much more!

Extra: 42 $J = 0$-states

$\Rightarrow$ don't show up in SUGRA!!

The big question

What are these states at large g_s?

- related to single centre black hole states?
- relation to AdS_2/CFT_1?