Multi-center Solutions from Kähler Manifolds

Nikolay Bobev

Simons Center for Geometry and Physics

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In collaboration with Ben Niehoff, Nick Warner, . . .

Outline

Overview

- BPS solutions of 5D ungauged supergravity are determined by a linear system of PDEs on a 4D hyper-Kähler base [Gauntlett-Gutowski-Hull-Pakis-Reall]. Explicit multi-center solutions constructed on a Gibbons-Hawking base [Denef], [Bena-Warner]. More general HK bases in 4D are hard to analyze explicitly [Bena-NB-Warner].
- Almost BPS solutions - Take the BPS ones and change orientation on the base [Goldstein-Katmadas], [Gimon-Larsen-Simón], [Bena-Dall'Agata-Giusto-Ruef-Warner]
- Non-BPS solutions - hard but one can make some progress! Use the "floating brane Ansatz" - lock the metric Ansatz to the electrostatic gauge potentials [Bena-Giusto-Ruef-Warner]. It encompasses the BPS and almost BPS solutions. The 4D base can be any electrovac solution. Many explicit examples constructed - Kerr, Kerr-Newman, Israel-Wilson [Bena-Giusto-Ruef-Warner], [NB-Ruef].
- A very general class of non-BPS solutions in 5D based on a class of 4D Kähler manifolds studied by LeBrun [NB-Niehoff-Warner].
- 6D BPS solutions were classified by GMR. Given as a 2D fibration over a 4D quaternionic almost-Kähler base. Only explicit examples so far have 4D HK bases. The LeBrun metrics lead to a large class of new examples [NB-Niehoff-Warner]....

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Floating Brane Ansatz

Finding generic non-BPS solutions in 5D is hard!

Interesting solution generating techniques if there are Killing vectors - inverse scattering, non-linear sigma model.

Use intuition from branes and try a different approach. Relax the HK condition on the base and look for solutions of EoM. To get a reasonable system use the "floating brane Ansatz" - the electrostatic potentials are related to the warp factors in the metric [Bena-Giusto-Ruef-Warner].

$$ds_5^2 = -(Z_1 Z_2 Z_3)^{-2/3}(dt + k)^2 + (Z_1 Z_2 Z_3)^{1/3} ds_4^2$$

$$A^{(I)} = -\frac{1}{Z_I}(dt + k) + B^{(I)} \,, \qquad \Theta^{(I)} = dB^{(I)} \,, \qquad I = 1, 2, 3$$

Floating Brane Ansatz

One ends up with a linear system of EoM on a 4D electrovac base.

$$R_{\mu\nu} = \frac{1}{2}\left(F_{\mu\rho} F_{\nu}^{\rho} - \frac{1}{4} g_{\mu\nu} F_{\rho\sigma} F^{\rho\sigma}\right), \qquad \text{where} \qquad F = \Theta^{(3)} - \omega_{-}^{(3)}.$$

$\Theta^{(3)}$ is self-dual, $\omega_{-}^{(3)}$ is anti-self-dual

$$\hat{\nabla}^2 Z_1 = \star_4(\Theta^{(2)} \wedge \Theta^{(3)}), \qquad (\Theta^{(2)} - \star_4 \Theta^{(2)}) = 2 Z_1\, \omega_{-}^{(3)},$$

$$\hat{\nabla}^2 Z_2 = \star_4(\Theta^{(1)} \wedge \Theta^{(3)}), \qquad (\Theta^{(1)} - \star_4 \Theta^{(1)}) = 2 Z_2\, \omega_{-}^{(3)},$$

$$\hat{\nabla}^2 Z_3 = \star_4[\Theta^{(1)} \wedge \Theta^{(2)} - \omega_{-}^{(3)} \wedge (dk - \star_4 dk)], \qquad dk + \star_4 dk = \frac{1}{2}\sum_{I=1}^{3} Z_I(\Theta^{(I)} + \star_4 \Theta^{I}).$$

Remove the bits in red and get back to the BPS system of equations!

LeBrun-Burns metrics

The LeBrun-Burns metrics are

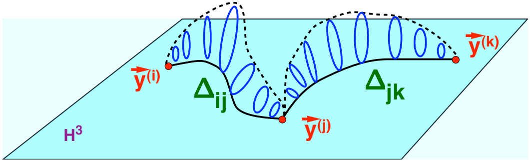
$$ds_4^2 = \zeta^2 \left[V^{-1}(d\tau + A)^2 + V \left(\frac{dx^2 + dy^2 + d\zeta^2}{\zeta^2} \right) \right] \quad (1)$$

with

$$\nabla^2_{\mathbb{H}_3} V = 0 \, , \qquad dA = \star_{\mathbb{H}_3} dV \quad (2)$$

This is very similar to Gibbons-Hawking but the 3D base is $\mathbb{H}_3$ instead of $\mathbb{R}^3$. Note that there is topology on which to put cohomological fluxes. The manifolds have $R = 0$ and can be made into electrovac solutions.

Rarely used in Physics [Atiyah-Witten]!



5D Asymptotics

There is an explicit solution for an arbitrary multi-center non-BPS solution with a LeBrun base [NB-Niehoff-Warner].

5D asymptotics - warped $AdS_2 \times S^3$. When the two angular momenta are equal we get the near horizon BMPV geometry.

$$ds_5^2 \approx -W_0(\theta)^{-2}\rho^4\left(dt - \frac{1}{2}(\beta_3 + 2\beta_1\beta_2)\frac{\cos^2\theta}{\rho^2}d\tau + \frac{\beta_3}{2}\frac{\sin^2\theta}{\rho^2}d\phi\right)^2$$
$$+ W_0(\theta)\left(\frac{d\rho^2}{\rho^2} + d\theta^2 + \cos^2\theta d\tau^2 + \sin^2\theta d\phi^2\right)$$

where

$$W_0(\theta) \equiv (\beta_1\beta_2(2\gamma + (\beta_1\beta_2 + \beta_3)\cos^2\theta))^{\frac{1}{3}}$$

for $\beta_3 = -\beta_1\beta_2$ we have $J_1 = J_2$ and the near horizon limit of the BMPV black hole.

6D BPS solutions

BPS solutions of 6D ungauged supergravity were classified by Gutowski-Martelli-Reall. Recently the underlying equations were showed to be linear [Bena-Giusto-Shigemori-Warner].

$$ds_6^2 = 2H^{-1}(dv+\beta)(du+\omega+\tfrac{1}{2}\mathcal{F}(dv+\beta)) - H ds_4^2$$

An important ingredient is a 4D base which is almost hyper-Kähler

$$dJ^{(A)} = \partial_v(\beta \wedge J^{(A)}) \,, \qquad i = 1,2,3$$

a trivial solution to this is a HK manifold. The 4D LeBrun metrics provide the first known non-trivial examples of solutions to this equation!

Moreover the 6D system of BPS equations on a LeBrun base is identical to the 5D non-BPS system coming from the floating brane Ansatz!

The explicit non-BPS solutions in 5D provide a new infinite family of 6D BPS solutions!

6D Asymptotics

In 6D we have two interesting asymptotics - a pp wave which was studied in GMR and rotating $AdS_3 \times S^3$ which is the near horizon limit of the BPS D1-D5-P black hole.

$$ds_6^2 \approx -2dv(du - \rho^2 dv + \rho \sin\theta d\phi) + d\rho^2 + \rho^2 d\Omega_3^2$$

$$ds_6^2 \approx -2\rho^2 dv \left(du - \frac{1}{\rho^2}(\sin^2\theta d\phi + \cos^2\theta d\chi) - \frac{2}{\rho^2} dv \right) + \frac{d\rho^2}{\rho^2} + d\Omega_3^2$$

where $d\Omega_3^2 = d\theta^2 + \sin^2\theta d\phi^2 + \cos^2\theta d\chi^2$

The explicit solutions on a LeBrun-Burns base are mutli-center solutions with or without horizons which are asymptotic to one of these two space-times.

In particular we have an infinite new class of regular solutions with (rotating) $AdS_3 \times S^3$ asymptotics. These should be understood in terms of the D1-D5-P CFT!

Open Questions

- The first examples of 6D BPS multi-center solutions beyond the Gibbons-Hawking class!
- What are these solutions dual to in the D1-D5-P CFT? Count them?
- Connection with the missing entropy from the recent work of Bena-Chowdhury-de Boer-El-Showk-Shigemori?
- Constructing more general solutions seems hard due to $dJ = \partial_v(\beta \wedge J)$. Understand this equation geometrically and find a class of 5D manifolds that obey it?
- Look at gauged supergravity and construct regular asymptotically AdS_5 solutions? The classification is known [Gauntlett-Gutowski], [Gutwoski-Reall].Technically more involved construction than the ungauged case. Only one example known. The 4D base is a quaternioninc-Kähler manifold with non-vanishing Ricci scalar. New solutions will be of obvious interest for AdS/CFT.

THANK YOU!