APPLIED
TRI-SASAKIAN REDUCTION

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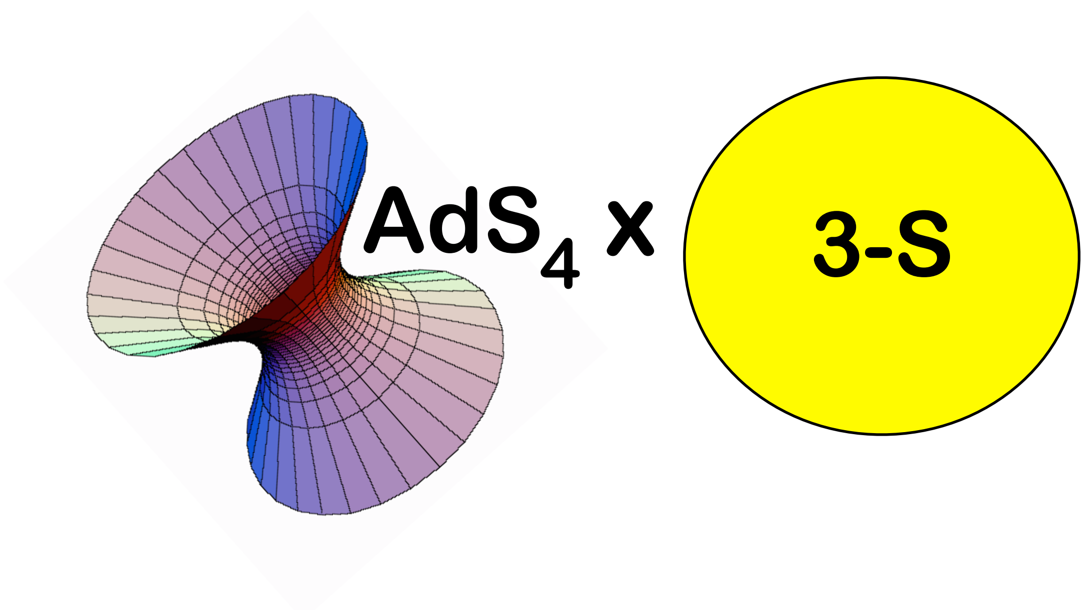
Based on arXiv:1110.5327 and work in progress, with P. Koerber
(+ previous work with A. F. Faedo and G. Dall'Agata)

tri-Sasakian AdS/CFT

7d 3-Sasakian ➜ N=3 AdS_4 vacua of 11D sugra



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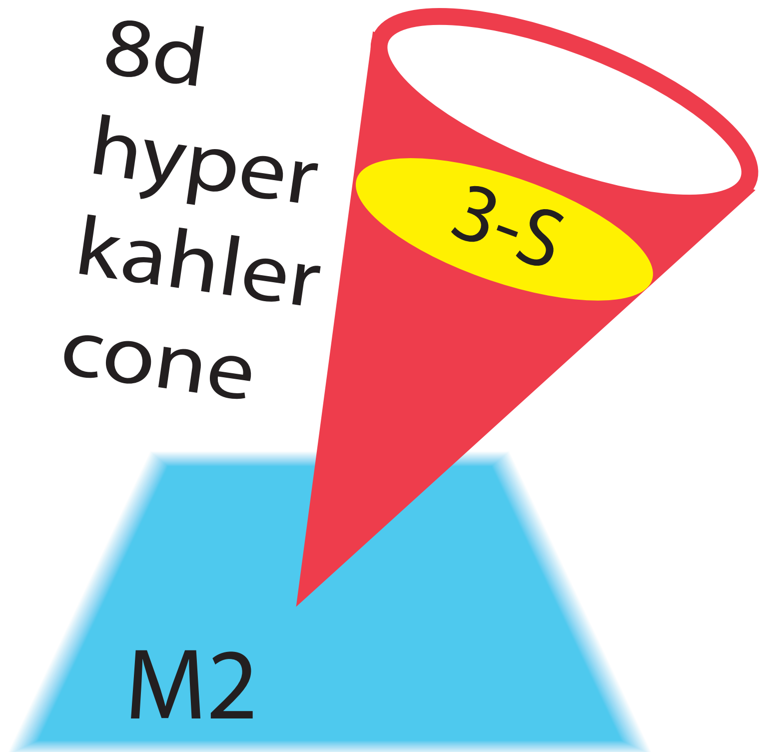

Examples :

- $S^7 = \frac{\mathrm{Sp}(2)}{\mathrm{Sp}(1)}$ • $N^{010} = \frac{\mathrm{SU}(3)}{\mathrm{U}(1)}$
- ∞ non-homogenous, $\mathrm{U}(1)\backslash\mathrm{U}(3)\,/\,\mathrm{U}(1)$

→ **universal truncation to gauged N=4 sugra**

see P. Koerber's talk

4d, N=4 supergravity

$$S = \frac{1}{2\kappa_4^2} \int (R_4 - 2\,V) * 1 + S_{\rm kin,scal} + S_{\rm kin,vec} + S_{\rm top}$$

full bosonic action

$$S_{\rm kin,vec} = \frac{1}{2\kappa_4^2} \int \, e^{V_1+V_2+V_3} \Big[- \frac{1}{2} e^{4U} g_{IJ} F_2^I \cdot F_2^J \; - \; \delta^{IJ} H_{2I} \cdot H_{2J}$$
$$- \frac{1}{2} \, e^{-4U} g^{IJ} \left(\mathcal{D} a_{1I} - 2\, c_{IK}\, \mathcal{D} c_1^K - c_{IL} c_K{}^L F_2^K - \epsilon_{IKL} a^L F_2^K \right) \cdot \left(\mathcal{D} a_{1J} - \ldots \right) \Big] *1$$

$$S_{\rm kin,scal} = \frac{1}{2\kappa_4^2} \int *1 \Big[- 12 ({\rm d}U)^2 - 4\, {\rm d}U \cdot {\rm d}(V_1+V_2+V_3) - \frac{1}{2} \left({\rm d}(V_1+V_2+V_3) \right)^2$$
$$- \frac{1}{4} D g_{IJ} \cdot D g_{KL}\, g^{IK} g^{JL} - e^{-4U} g^{IK} \delta^{JL} H_{1IJ} \cdot H_{1KL} - \frac{1}{2} \, e^{-2(V_1+V_2+V_3)} ({\rm d}\chi)^2$$
$$- \frac{1}{2} e^{-2(4U+V_1+V_2+V_3)} g_{IJ} \left(\mathcal{D} a^I + \epsilon^{I K_1 K_2} c_{K_1 K_3} D c_{K_2}{}^{K_3} \right) \cdot \left(\mathcal{D} a^J + \epsilon^{J L_1 L_2} c_{L_1 L_3} D c_{L_2}{}^{L_3} \right) \Big]$$

$$S_{\rm top} = - \frac{1}{2\kappa_4^2} \int \left[\chi \left(\mathcal{D} a_{1I} \wedge F_2^I + \mathcal{D} c_{1I} \wedge \mathcal{D} c_1^I \right) \; + \; 2\, \mathcal{D} \tilde{c}_1^I \wedge \mathcal{D} \tilde{a}_{1I} \right]$$

$$V = e^{-4U-3(V_1+V_2+V_3)} (e^{4V_1} + e^{4V_2} + e^{4V_3}) - 2\, e^{-4U-V_1-V_2-V_3} (e^{-2V_1} + e^{-2V_2} + e^{-2V_3})$$
$$- 24\, e^{-6U-V_1-V_2-V_3} + 2\, e^{-8U-V_1-V_2-V_3} (e^{2V_1} + e^{2V_2} + e^{2V_3}) + \; 4\, e^{-12U-V_1-V_2-V_3} \, ({\rm tr}\, c)^2$$
$$+ \; 2\, e^{-8U-3(V_1+V_2+V_3)} \left[(\chi + {\rm tr}\, c) \delta^{IK} - 2\, c^{(IK)} \right] g_{IJ} \delta_{KL} \left[(\chi + {\rm tr}\, c) \delta^{JL} - 2\, c^{(JL)} \right]$$
$$+ \; e^{-12U-3(V_1+V_2+V_3)} \left[2\, c^{IJ} c_{(IJ)} - ({\rm tr}\, c)^2 - 2\, \chi\, ({\rm tr}\, c) + 3\, k \right]^2 . \qquad (3.22)$$

consistent truncations:
solution generating tools

solutions for holography

AdS$_4$ vacua

Freund-Rubin
'round' tri-Sasakian
N=3

AdS_4 vacua

Freund-Rubin
'round' tri-Sasakian
N=3

Spontaneous
N=4 ➞ N=3
partial susy breaking

tri-Sasakian M_7
admit
4 global spinors
but
only 3 are Killing

AdS_4 vacua

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tri-Sasakian M_7
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but
only 3 are Killing

Field fluctuations organize in N=3 supermultiplets

1 massless gravity m.

1 massive gravitino m.

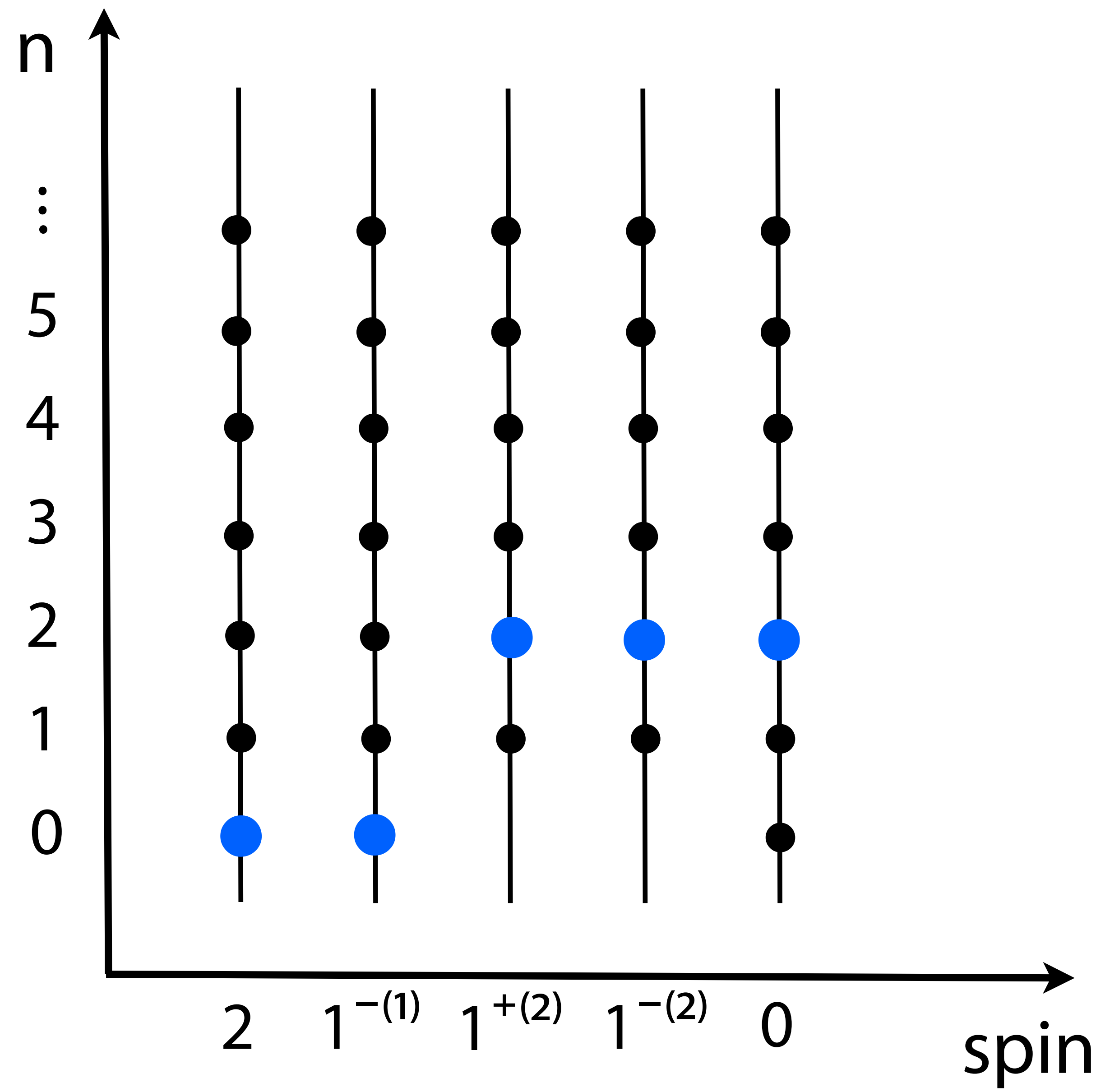
shadow multiplet

Billò, Fabbri, Frè,
Merlatti, Zaffaroni '00

does dual N=3 CFT see N=4 symmetry?

Comparison with S^7 spectrum

Bosonic mass spectrum around AdS_4 x round S^7



keep modes invariant under this Sp(2)

SO(8) ➜ Sp(2) x SO(3)

reduction on $\frac{\mathbf{Sp(2)}}{\mathbf{Sp(1)}} = S^7$

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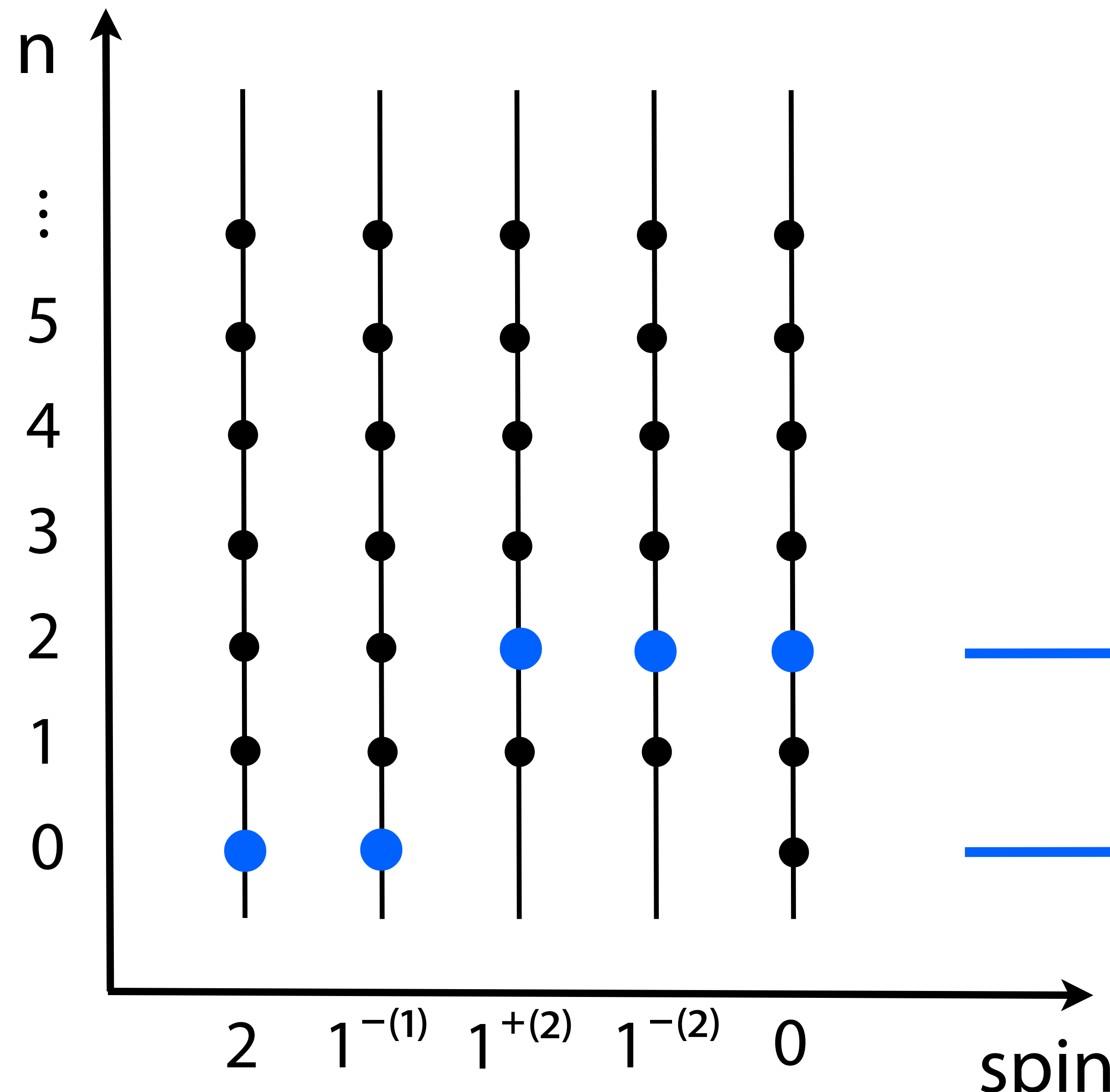
Bosonic mass spectrum around AdS_4 x round S^7



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→ N=3 massive gravitino mult.

→ N=3 massless gravity mult.

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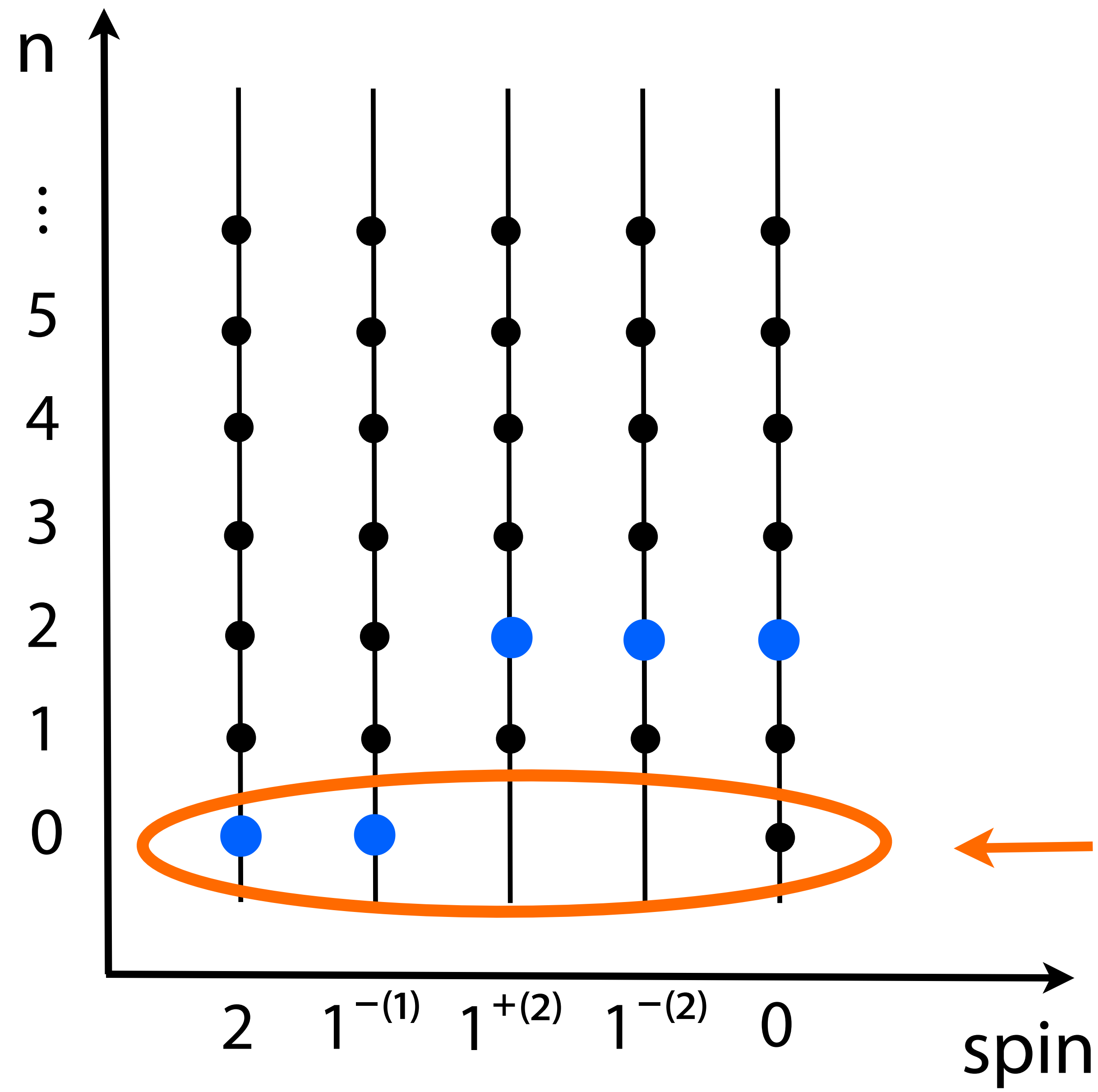
around AdS_4 x round S^7



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Our truncation
is not in N=8 sugra

4d, gauged SO(8) N=8 sugra

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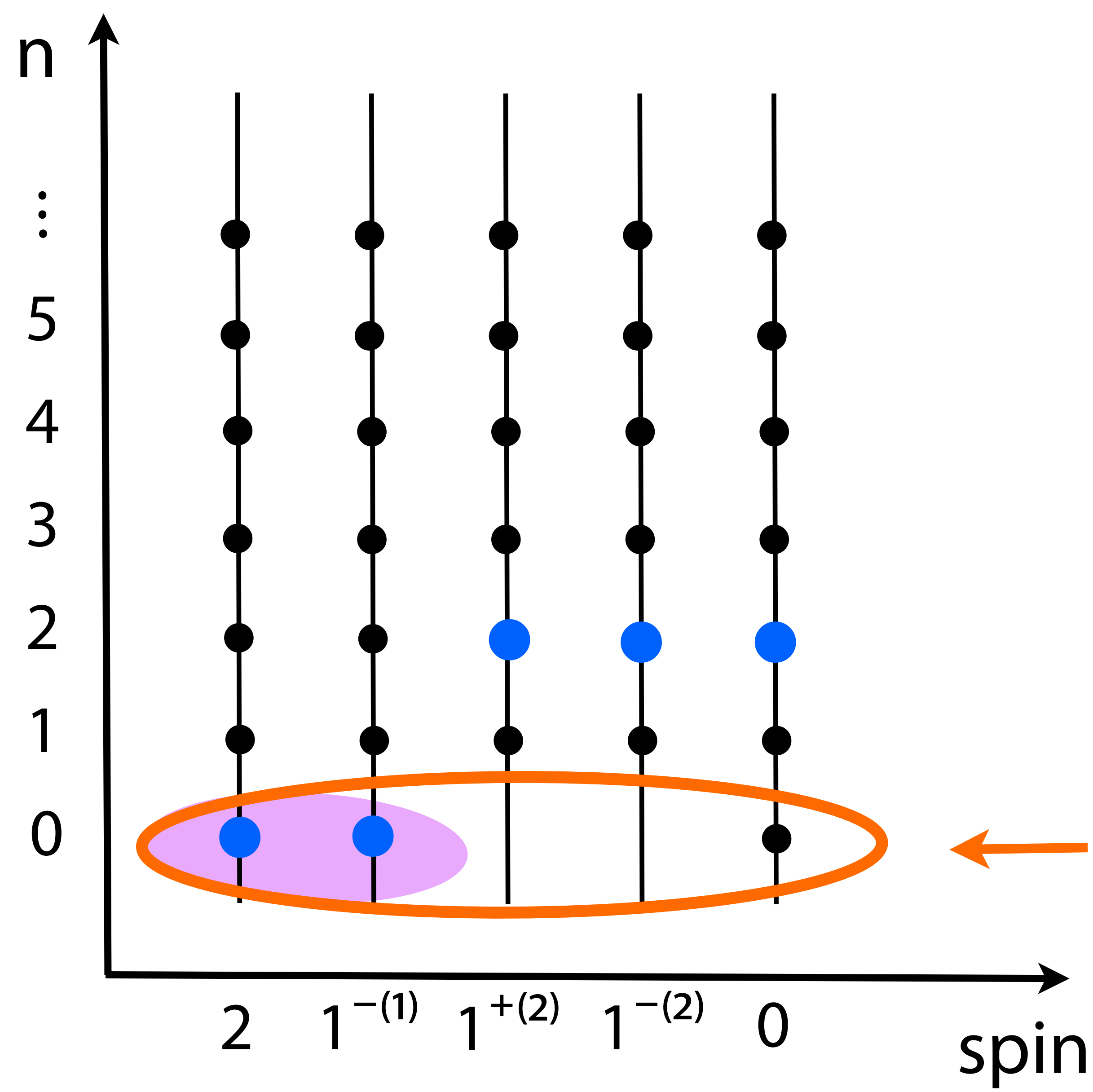

keep modes invariant under this Sp(2)

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Our truncation is not in N=8 sugra

4d, gauged SO(8) N=8 sugra

common subsector:
minimal N=3 gauged sugra
➜ consistent by itself

N=3 subtruncation

see also
Maldacena, Maoz

1 N=3 massless gravity m.

~~1 N=3 massive gravitino m.~~

N=3 subtruncation

see also Maldacena, Maoz

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minimal N=3 gauged supergravity

$SO(3)_R$ gauge group

$$S = \int d^4x \sqrt{|g|} \left(R - F^I_{\mu\nu} F^{I\,\mu\nu} + 24 \right)$$

N=3 subtruncation

see also
Maldacena, Maoz

1 N=3 massless gravity m.

~~1 N=3 massive gravitino m.~~

minimal N=3 gauged supergravity

$SO(3)_R$ gauge group

$$S = \int d^4x \sqrt{|g|} \left(R - F^I_{\mu\nu} F^{I\mu\nu} + 24\right)$$

- AdS/CMT: phase transitions with vector order parameter

  construct black holes with colorful horizon   Gubser

  p-wave superconductors ?   Gubser, Pufu

- Classify supersymmetric solutions

AdS$_4$ vacua

Freund-Rubin
'round' tri-Sasakian
N=4 ➞ N=3

AdS$_4$ vacua

Freund-Rubin
'round' tri-Sasakian
N=4 ➞N=3

Freund-Rubin
'squashed' tri-Sasakian
N=4 ➞N=1
not in SO(8) gauged sugra

+ skew-whiffed
N=0
stable

AdS_4 vacua

Freund-Rubin
'round' tri-Sasakian
N=4 → N=3

↓

Englert, $C_3 \neq 0$
N=0
unstable

Freund-Rubin
'squashed' tri-Sasakian
N=4 → N=1
not in SO(8) gauged sugra

↓

Englert, $C_3 \neq 0$
N=0
can be stable?

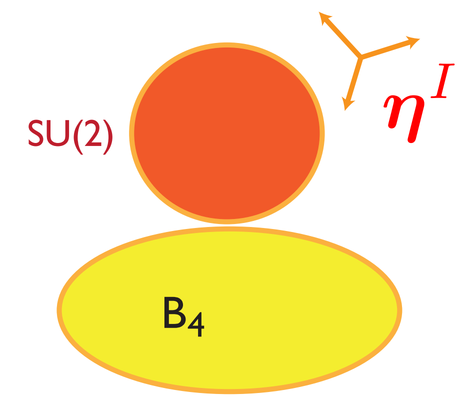
Pope-Warner
$C_3 \neq 0$, non-Einstein
N=0
unstable!

Bobev, Halmagyi, Pilch, Warner

N=4→N=1 subtruncation

restrict to SO(3)-invariant sector $\eta^I\,,\quad J^I$

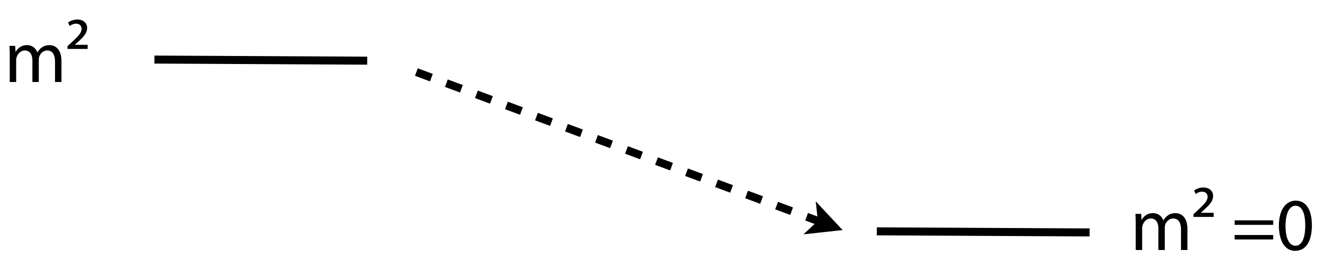
➜ throw away N=3 gravitini



'round' tri-Sasakian

'squashed' tri-Sasakian

N=4 ➞N=1



keeps massive gravitino

➜ massless in N=1 vacuum

Space Invaders scenario

N=1 subtruncation

minimal N=1 model — 2 chiral multiplets

not contained in SO(8) sugra

Kähler potential $\mathcal{K} = -\ln[i(\bar{z}_1 - z_1)] - 6\,\ln[i(\bar{z}_2 - z_2)]$

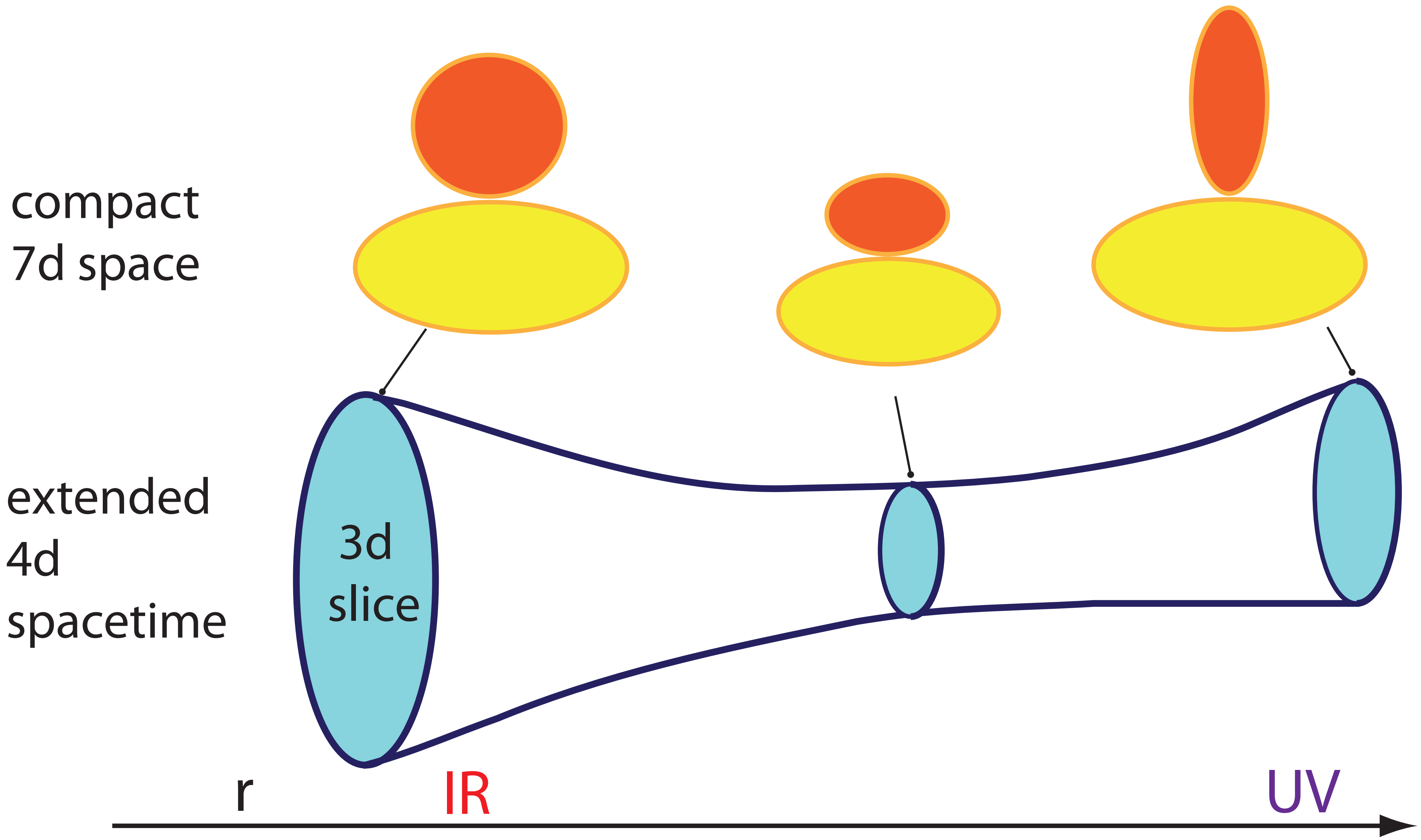
superpotential $\mathcal{W} = \,[z_2(z_2 + 2\,z_1) - k]$

'round' tri-Sasakian → 'squashed' tri-Sasakian

Interpolating flow

Ahn

Domain wall



Holographic RG flow ↔ Domain wall

RG flow between ABJM and Ooguri-Park theories

N=1 subtruncation

Interpolating flow

$$ds^2 \; = \; e^{2A(r)} ds^2 \, (\mathrm{Mink}_3) + dr^2$$

DW generated by first-order flow equations :

work in progress

warp : $A' \;=\; W$

scalars : $z^{x\prime} \;=\; -2\, g^{xy} \partial_y W$

Fake superpotential ?

$$V \;=\; 2 g^{xy} \partial_x W \partial_y W - 3W^2$$

Skenderis,Townsend

Conclusions

G-structures, coset spaces ➜ new consistent truncations
with massive modes (P. Koerber's talk)

- Map between lower-d sugra and string/M-theory
- Solutions deforming AdS ↔ breaking conformality
    - AdS / CMT : s-wave and p-wave superconductors
    - solutions with non-relativistic scaling symmetry (Schrödinger, Lifshitz)
    - RG flows AdS ↔ AdS, or AdS ↔ Lifshitz
    - construction of black-holes

... and many others!