# Coset Models as tools for a Microscopic Description of Extremal Kerr

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Based on work with
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# Coset models as solution generating techniques

**Assume $n-3$ Killing vectors, then**

$$\int L[g_{\mu\nu}, A^\alpha_\mu, \phi^i] = \int d^3x\sqrt{|g|}R + (coset)\left(\frac{G}{H}\right)$$

Solution determined by

- Base metric $ds^3$.
- Coset element $\nu \in G/H$.

More solutions are generated by acting with group element $G$:

$$\nu \to g(\nu) = \nu', \qquad g \in G$$

[Guica, Hartman, Song, Strominger, '08]

**Results so far:**

- Decoupling limit. Universal near-horizon geometry of extremal black holes.
- Chiral Virasoro algebra Universal asymptotic symmetry algebra extending $U(1)$ symmetry

# Extremal black hole / CFT correspondence (?)

**Conjecture:**

"The extremal Kerr black hole is described by a 2d CFT"

[Guica, Hartman, Song, Strominger, '08]

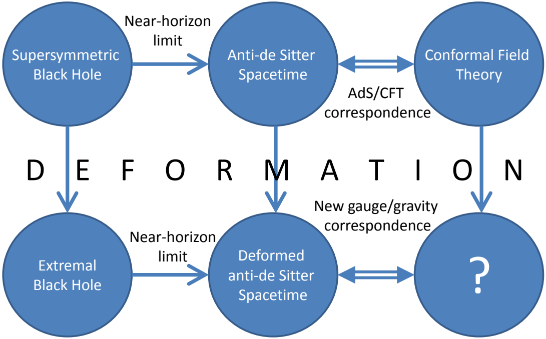
**Results so far:**

- Decoupling limit. Universal near-horizon geometry of extremal black holes.
- Chiral Virasoro algebra Universal asymptotic symmetry algebra extending $U(1)$ symmetry
- Current algebra Greybody factors in the near-horizon region = CFT 2pt correlators
- Ugly Non-extremal Hidden broken $SL(2,R) \times SL(2,R)$ symmetry in near black hole region.

# Is the extreme BH/CFT correspondence related to AdS/CFT?

**For black holes in $AdS_3$, one can use the boundary CFT:**
"Taking the near-horizon limit of the extremal BTZ near-horizon geometry corresponds to taking the DLCQ of the CFT"
[Balasubramanian, de Boer, Sheikh-Jabbari, Simon, '08]

**For general extremal black holes:**



# Several attempts

**The near-horizon geometry of some extremal black holes has been identified as**

- a deformation of the pinching orbifold of $AdS_3$ – defined in the limit of vanishing horizon area
  [Guica, Strominger, '10]
  The EVH/CFT correspondence [Sheikh-Jabbari, Yavartanoo, '11]
- a deformation of the null orbifold of $AdS_3$ – defined in the limit of vanishing horizon area
  [G.C., Song, Virmani, '11]
- locally equal to the near-horizon geometry of a non-rotating black string – obtained for the $5d$ singly rotating Myers-Perry BH
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- locally equal to the near-horizon geometry of a non-rotating black string – obtained for the 5$d$ singly rotating Myers-Perry BH
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# One attempt: relate extremal $Kerr \times S^1$ to $AdS_3 \times S^2$ which is the near-horizon geometry of M5[3] Branes

Consider the black string solution to $\mathcal{N} = 1$ minimal $5d$ supergravity corresponding to the brane intersection :

| | $t$ | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | $z$ | $r$ | $\theta$ | $\phi$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| M5 | × | — | — | × | × | × | × | × | | | |
| M5 | × | × | × | — | — | × | × | × | | | |
| M5 | × | × | × | × | × | — | — | × | | | |

It can be uplifted to $M$-theory on $T^6$. Dual CFT is "known". We add $4d$ angular momentum $J$, what happens?

Thanks to hidden symmetries, this solution can be found by using coset methods based on $\frac{G_{2(2)}}{SL(2,\mathbb{R})\times SL(2,\mathbb{R})}$ or $\frac{SO(4,4)}{SO(2,2)\times SO(2,2)}$.

# (a) Maldacena decoupling limit [Maldacena, 1998]

Express the solution in terms of $(n, J)$

$$Send \qquad l_p \rightarrow 0, \qquad r \rightarrow r\, l_p^3$$

This is a decoupling limit involving a near-horizon limit. The resulting geometry is

$$extremal\ BTZ \times S^2$$

Brown-Henneaux central charge and levels :

$$c_L = c_R = \frac{3l}{2G_3} = 6n^3, \qquad h_L = \frac{2J^2}{n^3}, \qquad h_R = 0$$

[Larsen, '98]
[G.C., de Buyl, Stotyn, Virmani, '10]

# (b) Corotating near-horizon decoupling limit

We introduce the overall scale $R > 0$ and the interpolating parameter $\Phi \in [0, \frac{\pi}{2}]$ defined by

$$R \sim \sqrt{n^2 + J^2}, \qquad \Phi \sim \arctan\left(\frac{J}{n}\right).$$

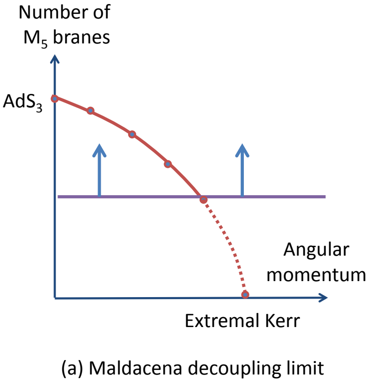
Then, the geometry scales as $R^2$ and reads as

$$\begin{aligned}\frac{ds^2}{R^2\, l_p^2} &= \Gamma(\theta)\left[-(k_\phi)^2 r^2 dt^2 + \frac{dr^2}{r^2} + d\theta^2\right] \\ &\quad + \gamma_{\phi\phi}(\theta) e_\phi^2 + 2\gamma_{\phi z}(\theta) e_\phi\, e_z + \gamma_{zz}(\theta) e_z^2\end{aligned}$$

where $e_\phi = d\phi + k_\phi r\, dt$, $e_z = dz + k_z r\, dt$.

At $\Phi = 0$, the solution is the null orbifold of $AdS_3$ (dual to the DLCQ CFT).
At $\Phi = \frac{\pi}{2}$, the solution is the extremal Kerr near-horizon geometry times $S^1$.

# Interpolation in the near-horizon limit




In case (b), the near-horizon geometry has a small curvature with respect to the Planck scale in the interpolation region between no angular momentum (described by the DLCQ CFT) and no $M_5$ brane charge (conjectured to be dual to a Kerr CFT).

# Conclusions

- It exists a supergravity solution interpolating between

$$\text{Susy Null orbifold of } AdS_3 \times S^2 \rightarrow NHEK \times S^1$$

- The dual theory to $AdS_3 \times S^2$ can be used to understand the interpolating geometry in perturbation theory around $AdS_3 \times S^2$.
- The dual description in terms of a DLCQ of a CFT is in line with the non-existence of dynamics in the extremal Kerr throat. [Amsel, Marolf, Horowitz, Roberts, '09]
- The irrelevant deformation is not understood. More work is needed to confirm / infirm the existence of the Kerr-CFT.
- So far, our picture is consistent with the Kerr-CFT conjecture.