Heating up the BIon

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Saclay, November 15, 2011

Based mainly on the papers
JHEP 1106: 058 (2011) and Nucl. Phys. B851: 262 (2011)
with Grignani, Marini, Obers and Orselli
Also based on the five blackfold papers written with Emparan, Niarchos and Obers
(and one of them with Rodriguez).

Motivation:

The blackfold method provides a new way to study D-brane probes in String Theory

In particular this means we have a new method to describe D-brane probes in thermal backgrounds

Two definitions of D-branes in string theory:

Open strings with Dirichlet boundary conditions (in some directions)
→ "D-branes are what open strings end on"

A Dp-brane source the Ramond-Ramond (RR) p+2 form field strength in the SUGRA effective low energy description of type IIA or IIB string theory

Polchinski: These two definitions define one and the same object

→ Open/closed string duality

Open string description of D-branes → DBI action

Low energy effective theory for a single D-brane is DBI action (valid for N=1, $g_s \ll 1$)

$$I_{\mathrm{DBI}} = -T_{\mathrm{Dp}} \int_{\mathrm{w.v.}} d^{p+1}\sigma \sqrt{-\det(\gamma_{ab} + 2\pi l_s^2 F_{ab})} + T_{\mathrm{Dp}} \int_{\mathrm{w.v.}} e^F \wedge C$$

Fradkin & Tseytlin.
Abouelsaood, Callan,
Nappi & Yost

Action not known in finite-temperature backgrounds (uses of DBI in such backgrounds neglects thermalization of the photons and superpartners – an issue which does not seem to be addressed in the literature)

Closed string description of D-branes → Blackfold approach

For $N \gg 1$ and $g_s N \gg 1$: D-branes backreact on the geometry (type IIA/B SUGRA)

Blackfold approach: Describes how to bend a D-brane probe along a submanifold

We can turn on F_{ab} by considering brane bound states, such as Dp-F1, Dp-D(p-2) etc.

The free energy of the brane configuration corresponds to the low energy effective action for the D-brane system, including all open string massive modes
→ Generalizes the DBI action

For T=0: The effective action is equivalent to the DBI action (expected from local SUSY)

For T > 0: A thermal effective action for D-branes!

Why is it important to understand D-brane probes in thermal backgrounds?

The D-brane probe a crucial tool for non-perturbative string theory phenomena
Relies on knowing the full non-linear D-brane dynamics

In this talk: Study the thermal generalization of the BIon solution of the DBI action

BIon solution: The first example of a solution of the DBI action that exploits the full non-linear dynamics → New phenomena introduced

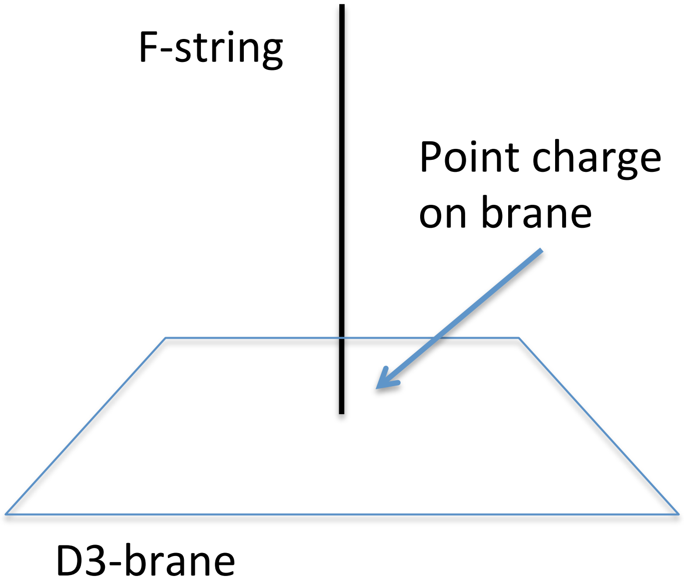
Heating up the BIon is a steppingstone towards more complicated cases of thermal D-brane probes (and other thermal brane probes as well)

Review of BIon solution of the DBI action

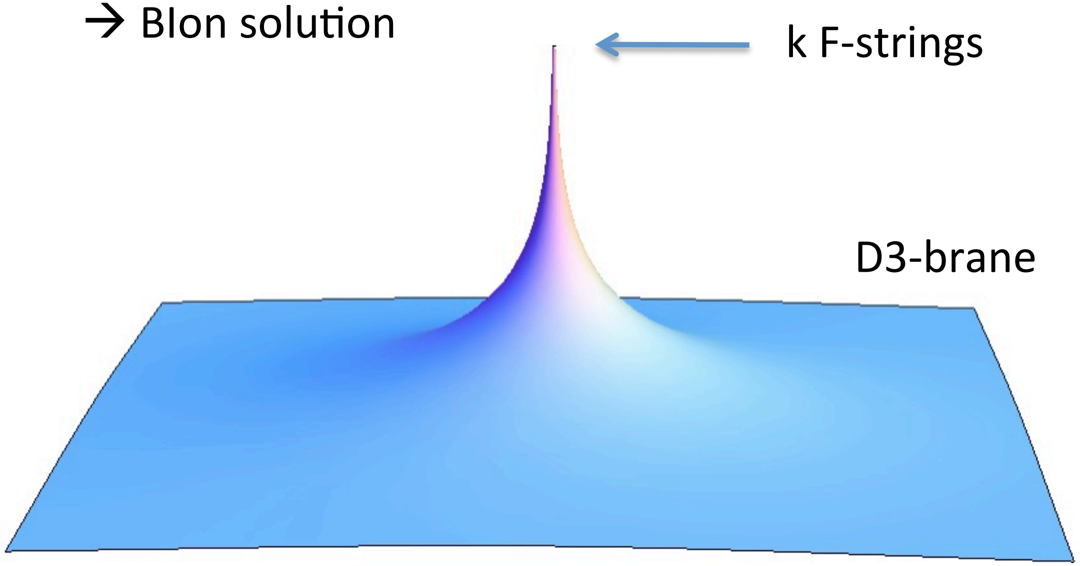
Callan & Maldacena. Gibbons. Emparan.

<u>Open strings ending on D3-branes</u>:

Linearized limit of DBI action:
Bosonic part is simply Maxwell's electrodynamics
(plus transverse scalars)



Full DBI: Non-linear electrodynamics, describes many coincident F-strings
→ BIon solution

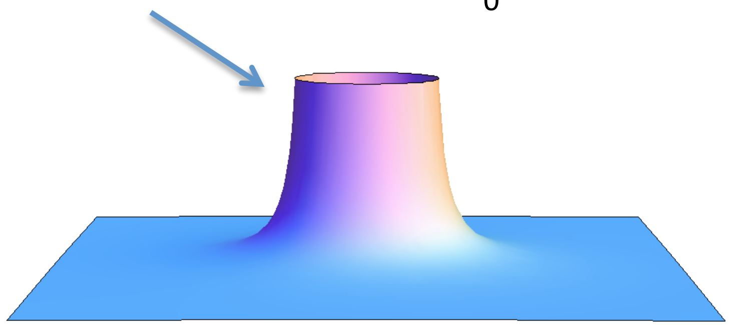


In the BIon solution the singular point at the point charge is resolved

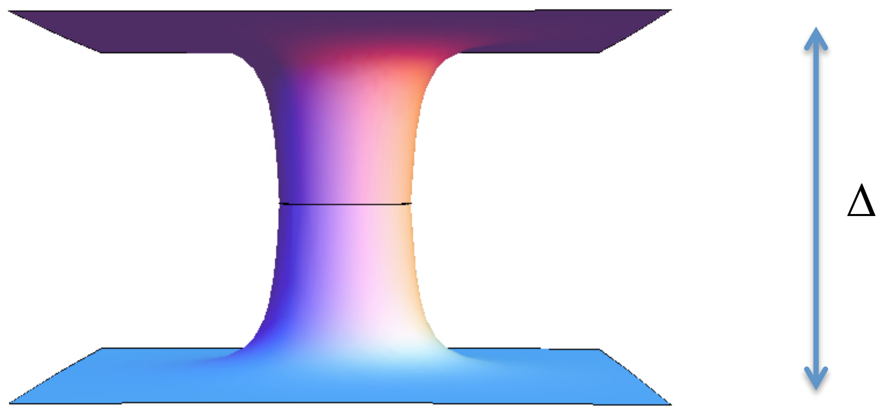
- The F-strings dissolve smoothly into the D-brane
- One can describe a large number of coincident F-strings
- F-strings described in terms of D-brane theory:
  k F-strings are "blown up" to a spherical D-brane

New phenomena from the non-linearity of DBI action

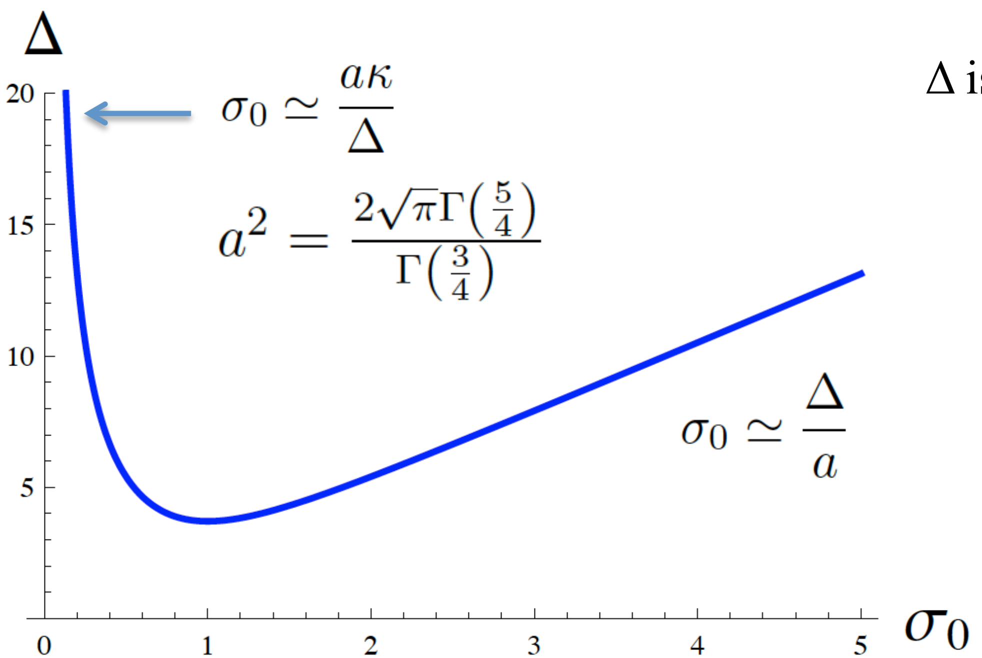
Also BIon solution with minimal S^2



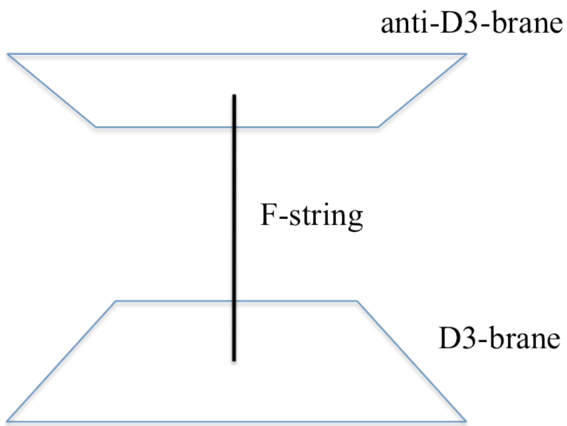
Need to attach a mirror of the solution
(otherwise D3-brane current not conserved)
→ We call it a brane-antibrane-wormhole solution



Δ is the distance between the D3-branes



In linearized limit:



How to heat up the BIon?

We want to put the D3-brane with electric flux in background of hot flat space

EOMs for all probe branes (no fluxes or dilaton):

$$K_{ab}{}^{\rho} T^{ab} = 0$$

Geometry of
brane worldsheet
embedding in
background space-time

Energy-momentum tensor

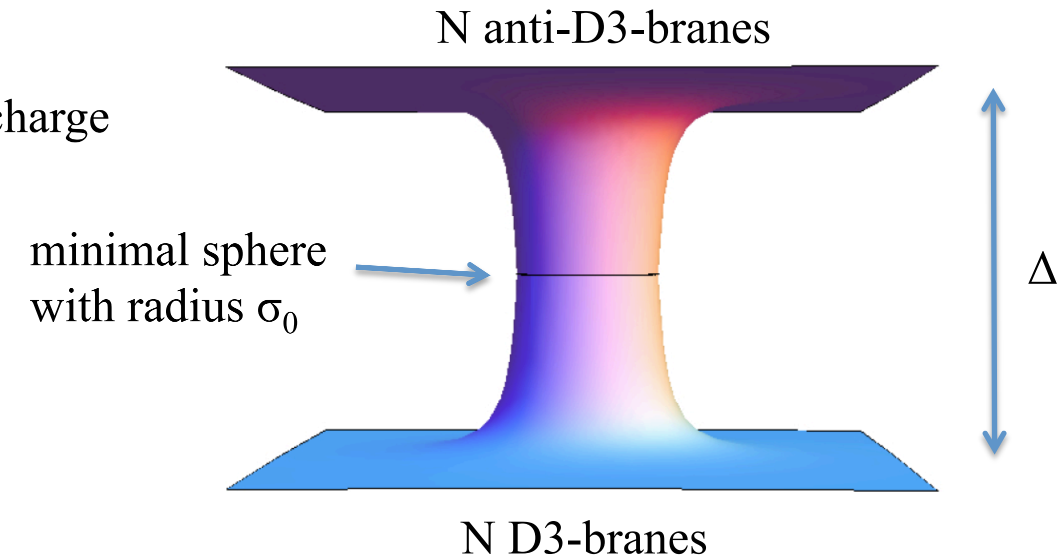
EOMs for DBI action also of this form, it is a generalization of the geodesic eq for branes

Our new approach: Replace T^{ab} with the energy-momentum tensor of the SUGRA solution for a black D3-F1 brane bound state

Corresponds to a finite-temperature D3-brane with electric flux and with $N \gg 1$ and $g_s N \gg 1$ (N = number of coincident D3-branes)

We switch from an open string description (the DBI action) to a closed string description of the D3-brane, via SUGRA solutions for the D3-brane

Configuration is N D3-branes
connected to N anti-D3-branes
through a throat with electric flux
corresponding to k units of F-string charge



Define: $\bar{T}^4 \equiv \frac{9\pi^2}{4\sqrt{3}} \frac{NT^4}{T_{D3}}$

We have the inequality: $\bar{T}^4 \sqrt{1 + \frac{\kappa^2}{\sigma^4}} \leq 1$

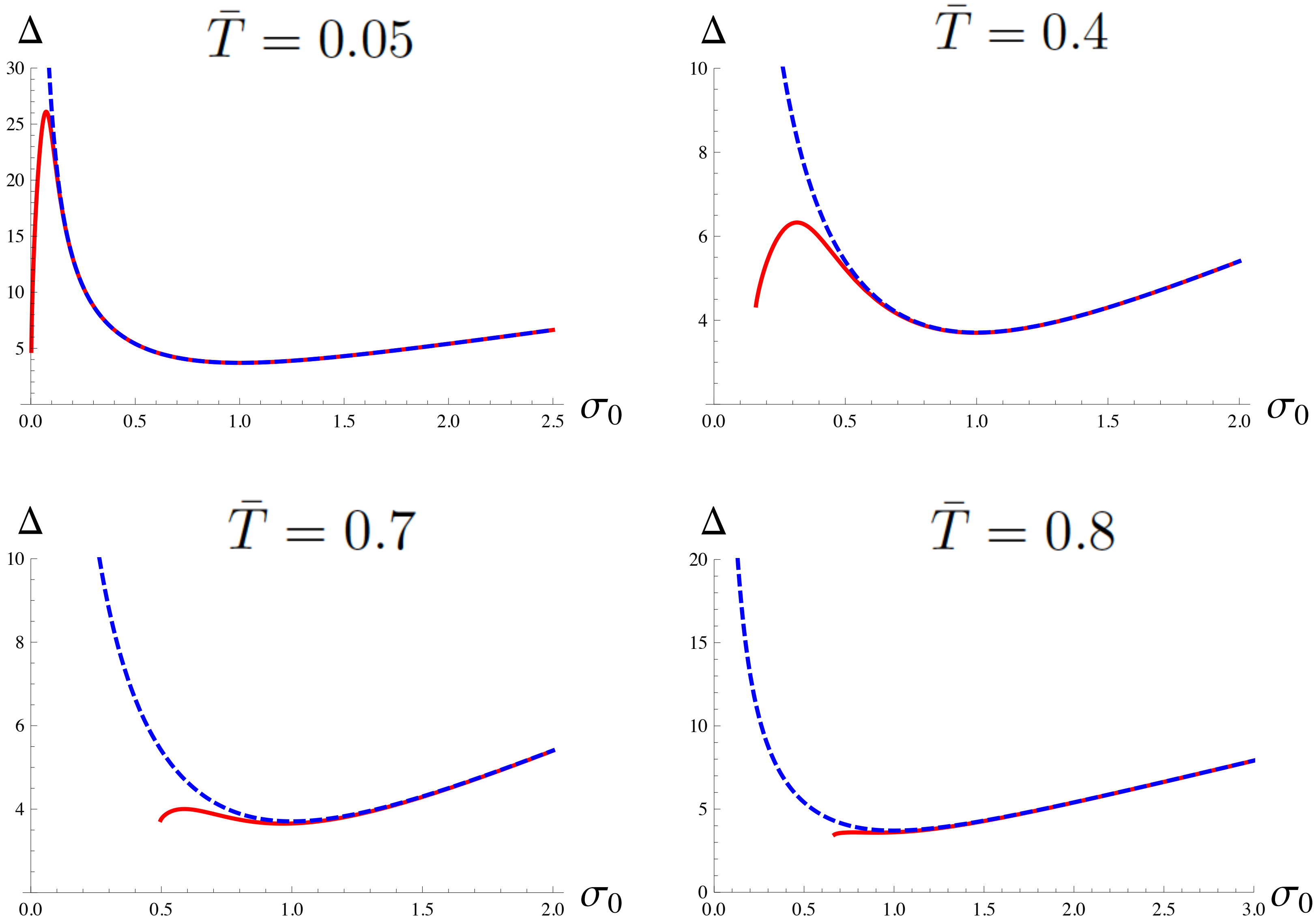
$\bar{T} = 1$ the maximal temperature of the black D3-brane

We also infer the bound: $\sigma_0 \geq \sigma_{\min} \equiv \frac{\sqrt{\kappa}\bar{T}^2}{(1 - \bar{T}^8)^{1/4}}$

The minimal sphere radius has a lower bound for a given temperature $\bar{T} > 0$

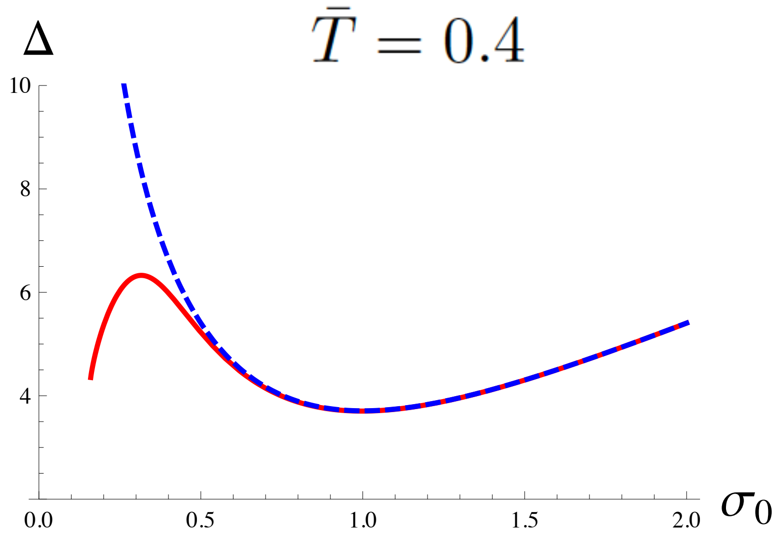
This is already a qualitative difference with the zero-temperature BIon
– there σ_0 can be arbitrarily small

We can now make σ_0-Δ diagrams for various values of $\bar{T} \leq 1$ (we set κ=1)



Blue line is the zero-temperature curve

We see that:
1)Not possible to have $z(\sigma_0) = \infty$ for $\sigma_0 < \infty$
2)Three branches instead of two
3)Local maximum for Δ
Goes like $T^{-2/3}$ for small T



Thus, we conclude

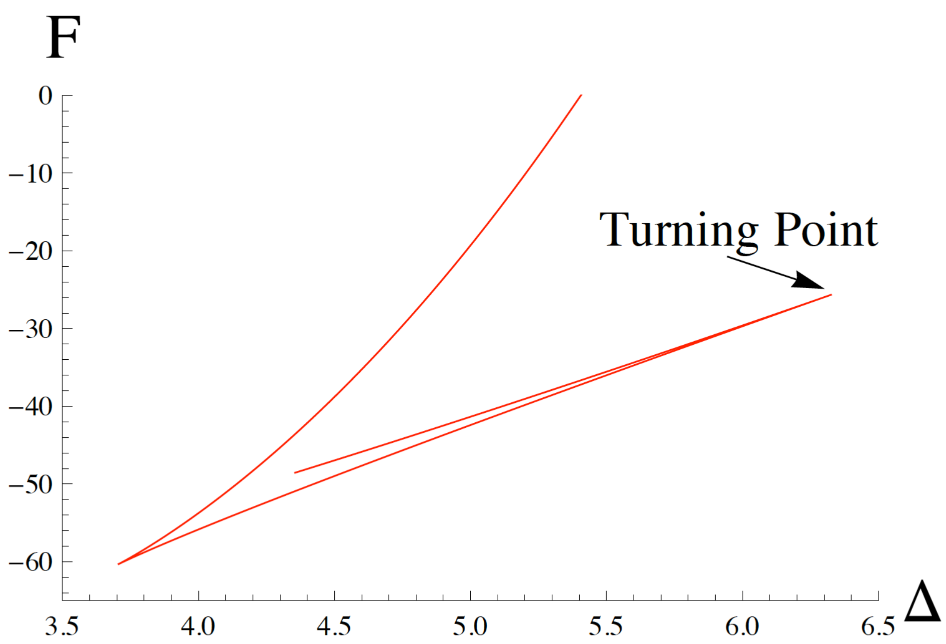
I) Non-zero temperature BIon qualitatively different than zero temperature BIon, even for small T

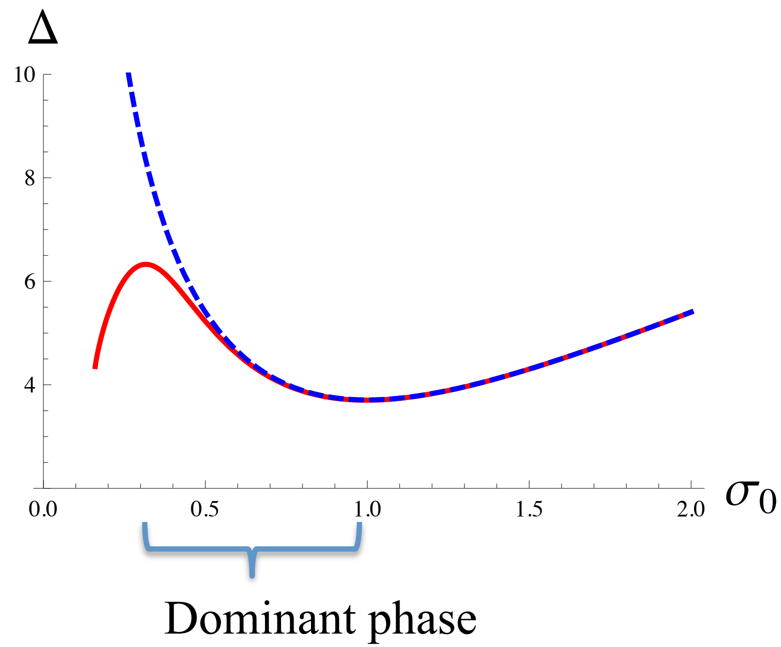
Note here that the "Euclidean DBI probe" method would have given the "blue curve", i.e. the same σ_0-Δ diagram as the zero temperature BIon

II) There is no immediate generalization of the infinite spike BIon solution to non-zero temperature

Comparison of phases: Dominant phase is the one with least free energy

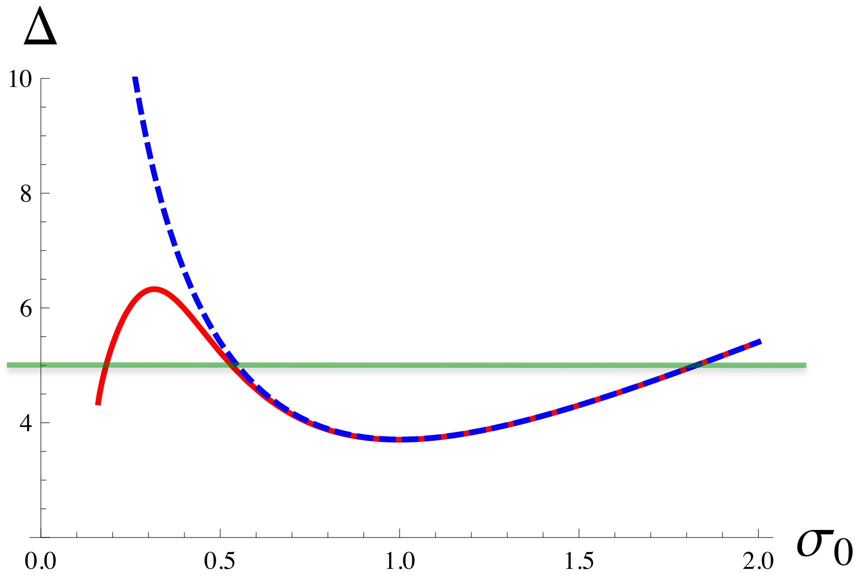
Example: $\bar{T} = 0.4$

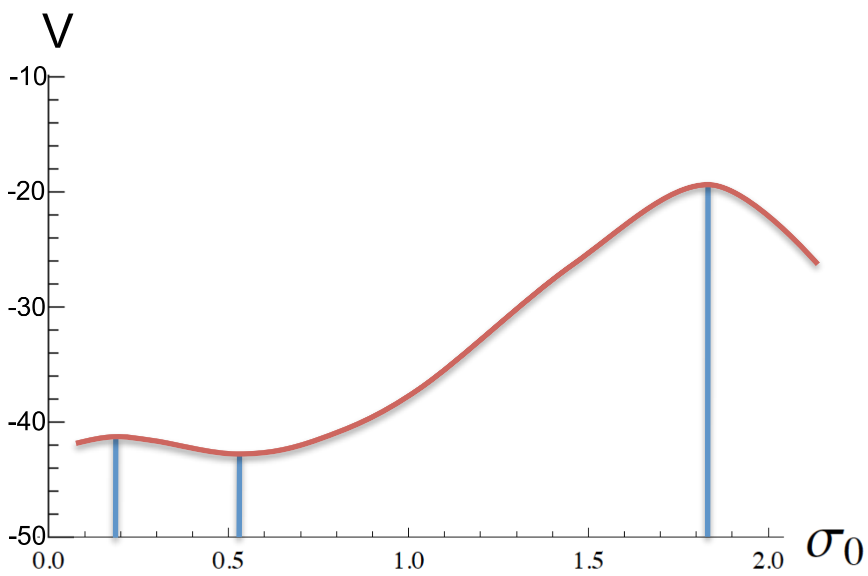




Dynamics of phases

Consider for given temperature $\bar{T} = 0.4$ and separation $\Delta = 5$:





Extrema of potential: The static solutions, value of potential set to free energy value

Decay channels of unstable equilibria:

Increasing σ_0: Time dependent solution with growing σ_0, everything vanishes

Decreasing σ_0: Either the dominant solution with minimal free energy
Or σ_0 decreases further towards zero, in case the F-string charge annihilate, leaving the brane and antibrane without the wormhole

Conclusions:

- New method for D-brane probes in thermal backgrounds

  Previous method using classical Wick rotated DBI action is inaccurate

- Method employs blackfold technique of making probe brane with EM tensor read off from SUGRA branes

  Different regime than DBI action

  DBI: $N = 1$ and $\lambda \ll 1$
  Blackfold: $N \gg 1$ and $g_s N \gg 1$

  Finite temperature physics found classically
  since we use closed string description

- We found thermal generalization of BIon solution to hot flat space

  Qualitatively different than the zero temperature case
  Shows explicitly that we get different results using the new method

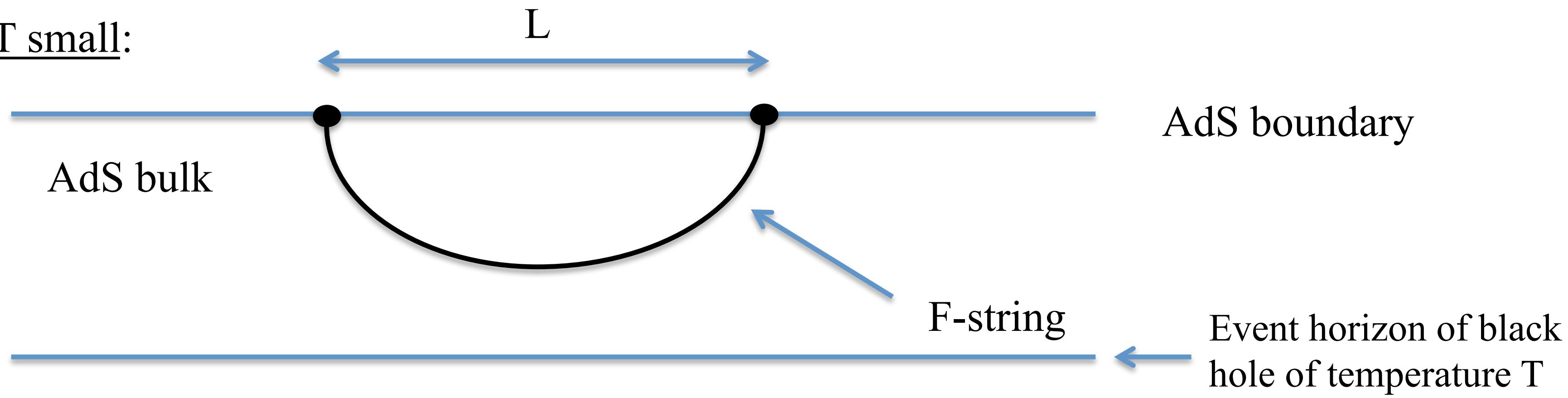
  Interesting new thermal physics, and also new dynamics

  Would be interesting to know qualitative arguments for the three equilibria,
  in particular the new equilibrium for $T > 0$

Outlook:

We can reconsider quark-antiquark system with AdS/CFT (here CFT in deconfined phase)

<u>LT small</u>:



Previous method: Classical Nambu-Goto action for F-string

We use SUGRA F-string → Accurate description for T > 0

Important issue in black hole background: **The Tolman Law**

$$T_{\rm local} = \frac{T}{R_0}$$

T: Temperature at infinity

T_{local}: Local temperature

R_0: The local red-shift

Probe brane should locally be in thermal equilibrium with T_{local}

But near a black hole: $R_0 \to 0$

Thus, a probe brane (in a static embedding of the background) is heated up to arbitrarily large temperatures near a black hole horizon!

THE END!

What is the blackfold method? - A quick reminder

Think of a black p-brane of radius r_0 (thickness)

- Allow the brane to bend, let R_b be the length scale of that meaning that for distances << R_b brane approximately flat
- Allow that it goes out of thermal equilibrium with length scale R_e i.e. for distances << R_e brane is in thermal equilibrium
- Put the brane in a curved background of length scale R_c i.e. for distances << R_c the background is flat space

Blackfold: Effective description in the regime $r_0 \ll R_b, R_e, R_c$

We "integrate out" the small scale DOFs connected to the r_0 scale
→ Gives an effective fluid energy-momentum tensor T_{ab} on the brane

Long-distance dynamics: Conservation of the energy-momentum tensor T_{ab}
→ This gives the equations of motion for the brane plus the conservation of the energy-momentum located on the brane

Niels told you about applications to find neutral black holes and Joan about response and transport coefficients

In this talk we consider an application of the Blackfold approach to String Theory

Validity of the brane probe approximation (in blackfold formalism):

Thickness scale of blackfold is roughly the charge radius r_c, which is the characteristic length scale of the brane geometry associated with the D3-brane charge for the extremal brane
(since we are in the branch connected to the extremal solution)

Charge radius r_c:

$$r_{\rm c}^4 \sim \frac{N}{T_{\rm D3}} \sqrt{1 + \frac{\kappa^2}{\sigma^4}}$$

Brane probe description valid for whole solution for given T, N, k and σ_0 when:

1) The brane thickness scale is much smaller than the radius of the minimal sphere:
$r_c(\sigma_0) << \sigma_0$

2) The brane thickness scale is much smaller than the length scale of the curvature of the embedding:
$r_c(\sigma_0) << L_{curv}(\sigma_0) \sim \sigma_0$

Can one observe the new thermal physics in the limit $\bar{T} \rightarrow 0$?

Put for simplicity $N_{D3} = 1$ (like for DBI probe)

Perturbative corrections to a D3-brane EM tensor small for $\bar{T} << 1$: $\bar{T} \sim g_s^{\frac{1}{4}} l_s T$

But we have

$$\frac{\Delta_{\text{min}}}{l_s} \sim g_s^{\frac{1}{2}} k^{\frac{1}{2}}$$ ← Comes from perturbative part

$$\frac{\Delta_{\text{max}}}{l_s} \sim g_s^{\frac{1}{3}} k^{\frac{1}{2}} (l_s T)^{-\frac{2}{3}}$$ ← Comes from non-perturbative part

We can find regimes where Δ_{max} survives while Δ_{min} goes to zero

Specifically for $g_s = \lambda/N$ (AdS/CFT dictionary, N is from background flux, $R^4 = \lambda l_s^4$):

$$\bar{T} \sim N^{-\frac{1}{4}} RT \qquad \frac{\Delta_{\text{min}}}{R} \sim k^{\frac{1}{2}} N^{-\frac{1}{2}} \lambda^{\frac{1}{4}} \qquad \frac{\Delta_{\text{max}}}{R} \sim k^{\frac{1}{2}} N^{-\frac{1}{3}} \lambda^{\frac{1}{4}} (RT)^{-\frac{2}{3}}$$

→ Δ_{max} can scale like N^0 given the appropriate scaling of k with N

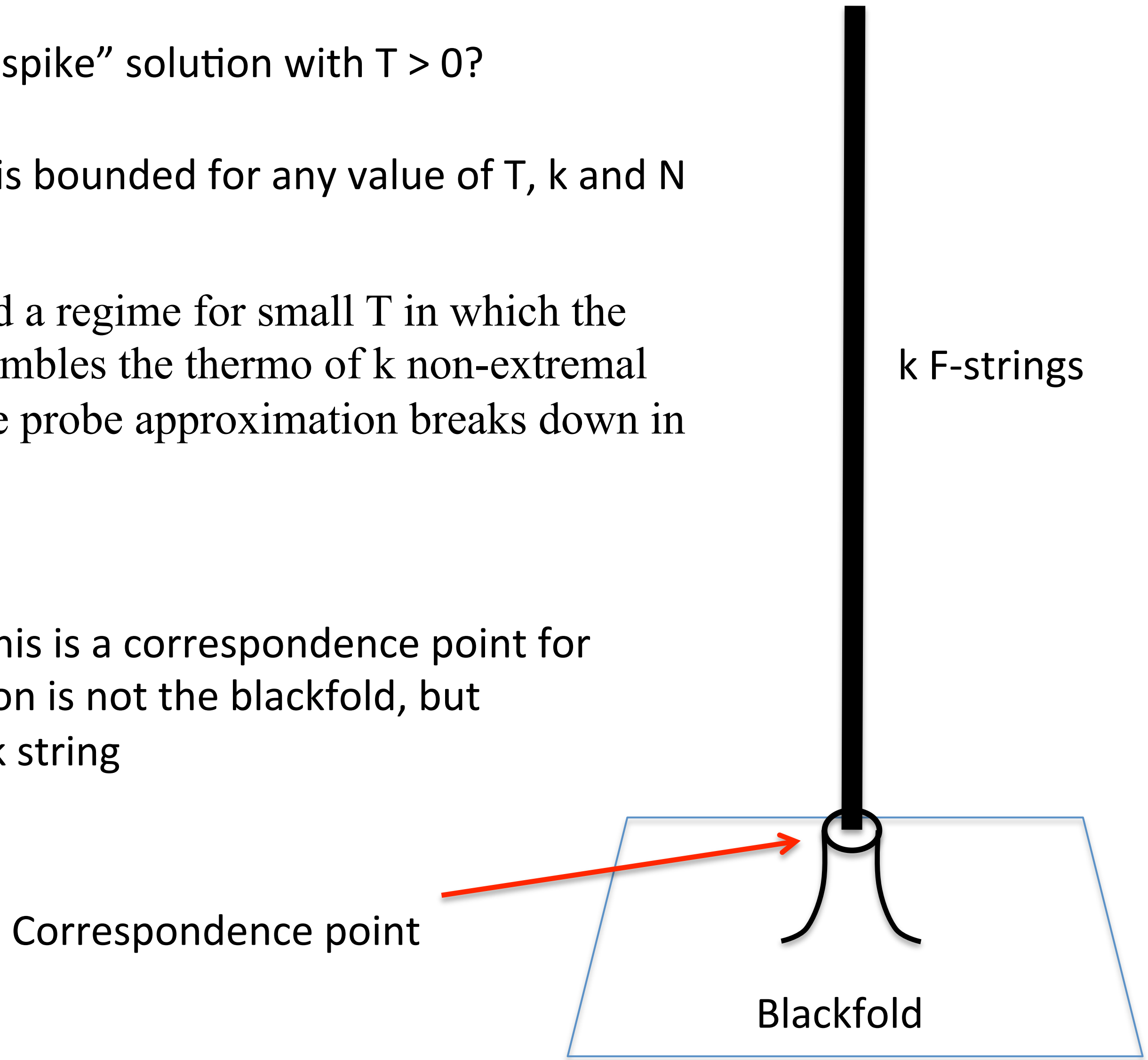
Generalization of the infinite spike solution for non-zero temperature?

Can we have an “infinite spike” solution with T > 0?

At first sight no, since Δ is bounded for any value of T, k and N

However, if we try to find a regime for small T in which the local thermo at $\sigma=\sigma_0$ resembles the thermo of k non-extremal F-strings, we find that the probe approximation breaks down in this regime

Natural to suggest that this is a correspondence point for which the right description is not the blackfold, but instead the straight black string



New extremal (and non-extremal) black holes in String Theory

As Niels reviewed: You can use the blackfold method to bend black p-branes along p-dimensional space-like submanifolds

→ Gives rise to stationary black hole solutions

In particular one can wrap branes on products of spheres/circles:

$$S^{n_1} \times S^{n_2} \times \cdots \times S^{n_k}$$

One finds equilibrium configurations by spinning along the spheres/circles
→ This balances the force of tension with the centrifugal force

One can now use the charged p-branes of string theory: Dp-branes, F-strings, NS5-branes and Mp-branes

Examples:

10D type IIA/B SUGRA

		Worldvolume	$\perp$ Sphere
	F1-D1	(helical) S^1	s^7
D0-D2	F1-D2	$\mathbb{T}^2$	s^6
	F1-D3	S^3 , $\mathbb{T}^3$	s^5
D0-D4	F1-D4	$S^3 \times S^1$, $\mathbb{T}^4$	s^4
	F1-D5	S^5 , $S^3 \times \mathbb{T}^2$	s^3
D0-D6	F1-D6	$S^3 \times S^3$, $S^5 \times S^1$	s^2

New black holes made with D(p-2)-Dp and F1-Dp brane bound states

Taking the extremal limit one should find new extremal configurations in String Theory
→ Generalizing supertubes

Should be possible to find these new configurations using the DBI action as well

But can we also find new extremal black holes, i.e. with non-zero horizon area? Yes!

Take D1-D5-P brane bound state, extremal limit of this has non-zero horizon area

We find the following new configurations using the D1-D5-P brane bound state:

Dimension (non-compact)	Worldvolume	$\perp$ Sphere
$D = 10$	S^5 , $S^3 \times \mathbb{T}^2$	s^3
$D = 9$	$S^3 \times S^1$, $\mathbb{T}^4$	s^3
$D = 8$	S^3 , $\mathbb{T}^3$	s^3
$D = 7$	$\mathbb{T}^2$	s^3
$D = 6$	S^1	s^3

However, no new SUSY configurations → The brane charges are not global and hence cannot appear in a BPS formula