Covariantising rotating black holes

Stefanos Katmadas

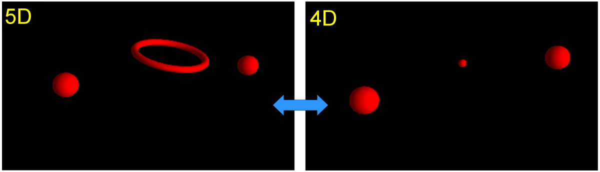
CPHT Paris

November 17, 2011

The 4D/5D story

The 4D/5D connection has been fruitful in the study of BH's in SUGRA:

- In four dimensions all BHs have spherical horizon topology
- They lift to two versions of black objects in 5D: familiar black holes and black rings with an $S^2 \times S^1$ horizon



Lift to 5D

The theory

Consider supergravity in 5 dimensions coupled to a set of vector multiplets, labeled by I: $(\sigma^I, A^I_\mu, \dots ; \Omega^{i\,I})$

- The most important two derivative bosonic terms in the action are:

$$\mathcal{L}_2 \sim \mathcal{R} + C_{IJK}\, e^{-1} \varepsilon^{\mu\nu\rho\sigma\tau} A_\mu{}^I F_{\nu\rho}{}^J F_{\sigma\tau}{}^K + C_{IJK}\sigma^I \left[\tfrac{1}{2}\mathcal{D}_\mu \sigma^J\, \mathcal{D}^\mu \sigma^K + \tfrac{1}{4} F^J \cdot F^K\right] + \dots$$

Invariant under $F \to -F$ if the **spacetime orientation** is flipped to compensate for the CS term $\Rightarrow$
Twist the 5D BPS equations to construct non-BPS solutions

non-BPS solutions

- This produces "almost-BPS", "floating brane" equations and more... [Goldstein, S.K., Bena, Warner, Ruef et al., 2008-...].
- These equations describe a rather rich space of multi-centre solutions, including non-extremal ones (!)
- Further extension of these results has been made possible by reduction to 3D [Bossard & Ruef, 2011].

non-BPS solutions

- This produces "almost-BPS", "floating brane" equations and more... [Goldstein, S.K., Bena, Warner, Ruef et al., 2008-...].
- These equations describe a rather rich space of multi-centre solutions, including non-extremal ones (!)
- Further extension of these results has been made possible by reduction to 3D [Bossard & Ruef, 2011].

The equation for the 4D angular momentum singles out the KK vector:

$$\nabla \times \vec{\omega} = \nabla(H^0\,\omega_5) + \dots$$

In 4D, a new symmetry of the e.o.m. arises: **electric/magnetic duality**
Can we use e/m duality to get more insight of the generic solution?

Back to 4D

In 4D, electric/magnetic duality acts as follows:

$$\mathcal{F} = \begin{pmatrix} F^A \\ G_B \end{pmatrix} \rightarrow \begin{pmatrix} U^A{}_C & Z^{AD} \\ W_{BC} & V_B{}^D \end{pmatrix} \begin{pmatrix} F^C \\ G_D \end{pmatrix} , \quad \text{where} \quad G_{\mu\nu\, A} \sim \frac{\partial \mathcal{L}}{\partial F^A_{\mu\nu}} .$$

In $\mathcal{N} = 2$ theories the scalars are part of the vector multiplets, and transform under symplectic reparameterisations.

$$\Omega = \begin{pmatrix} X^A \\ F_A \end{pmatrix} , \quad \text{with } X^I \sim \sigma^I + \mathrm{i} A^I_\psi , \text{ and } X^0 \text{ the KK scalar} .$$

All scalar dependent quantities are given in terms of a holomorphic prepotential

$$F \sim \frac{C_{IJK} X^I X^J X^K}{X^0} .$$

Seed solution

A pair of static BPS/non-BPS solutions are given by

$$ds^2 = -\mathrm{e}^{2U}(dt \qquad)^2 + \mathrm{e}^{-2U} d\vec{x}\,,$$

$$\mathcal{F} = \star\mathrm{d}\mathcal{H}_{\pm}\,, \qquad 2\,\mathrm{e}^{-U}\,\mathrm{Im}\,\Omega = \mathcal{H}_{+}\,,$$

$$\mathcal{H}_{\pm} = (\pm H^0,\, 0\,;\, 0,\, H_I)\,.$$

The metric and harmonic functions are given by:

$$\mathrm{e}^{-2U} = \sqrt{H^0 C^{IJK} H_I H_J H_K}\,,$$

$$H^0 = h^0 + \frac{p^0}{r}\,, \quad H_I = h_I + \frac{q_I}{r}\,,$$

Seed solution

The rotating seed solution carrying the same charges is [Bena et al. 2009]

$$ds^2 = -e^{2U}(dt+\omega)^2 + e^{-2U}d\vec{x}\,,$$
$$\mathcal{F} = \star \mathbf{d}\mathcal{H}_- - 2\,\mathbf{d}(e^U \mathrm{Re}\,\Omega\,\omega)\,, \quad 2\,e^{-U}\,\mathrm{Im}\,\Omega = \mathcal{H}_+ + \mathcal{R}\,,$$
$$\mathcal{H}_\pm = (\pm H^0,\,0\,;\,0,\,H_I)\,, \quad \mathcal{R} = \left(0,\,0\,;\,-\tfrac{M}{H^0},\,0\right).$$

The metric and harmonic functions are given by:

$$e^{-2U} = \sqrt{H^0 C^{IJK} H_I H_J H_K - M^2}\,, \quad \star\mathbf{d}\omega = \mathbf{d}M = \mathbf{d}\langle\mathcal{H},\mathcal{R}\rangle\,,$$
$$H^0 = h^0 + \frac{p^0}{r}\,, \quad H_I = h_I + \frac{q_I}{r}\,, \quad M = b + \frac{J\cos\theta}{r^2}\,,$$

Seed solution

The rotating seed solution carrying the same charges is [Bena et al. 2009]

$$ds^2 = -e^{2U}(dt+\omega)^2 + e^{-2U}d\vec{x}\,,$$
$$\mathcal{F} = \star \mathbf{d}\mathcal{H}_- - 2\,\mathbf{d}(e^{U}\mathrm{Re}\,\Omega\,\omega)\,, \quad 2\,e^{-U}\,\mathrm{Im}\,\Omega = \mathcal{H}_+ + \mathcal{R}\,,$$
$$\mathcal{H}_\pm = (\pm H^0,\, 0\,;\, 0,\, H_I)\,, \quad \mathcal{R} = \left(0,\, 0\,;\, -\tfrac{M}{H^0},\, 0\right).$$

The metric and harmonic functions are given by:

$$e^{-2U} = \sqrt{H^0 C^{IJK} H_I H_J H_K - M^2}\,, \quad \star\mathbf{d}\omega = \mathbf{d}M = \mathbf{d}\langle\mathcal{H},\mathcal{R}\rangle\,,$$
$$H^0 = h^0 + \frac{p^0}{r}\,, \quad H_I = h_I + \frac{q_I}{r}\,, \quad M = b + \frac{J\cos\theta}{r^2}\,,$$

- Applying duality rotations results to a generic ratio of harmonics
- The solution above is static in the limit $J=0$, but it nevertheless contains a nontrivial $\mathcal{R} \propto b/H^0$ [Cardoso et al., Gimon et al., 2007]

BPS vs non-BPS

Based on the above we proposed a generic ansatz for extremal under-rotating BHs, encompassing all known solutions [Galli et al. 2010]

$$2\,\mathrm{e}^{-U}\,\mathrm{Im}\,\mathrm{e}^{-\mathrm{i}\alpha}\Omega = \mathcal{H} + \mathcal{R}\,,$$

where $\mathcal{H}$ are harmonic functions **replacing the scalar attractor values**

- The angular momentum harmonic is the invariant $M = \langle \mathcal{H}, \mathcal{R} \rangle$

BPS vs non-BPS

Based on the above we proposed a generic ansatz for extremal under-rotating BHs, encompassing all known solutions [Galli et al. 2010]

$$2\,\mathrm{e}^{-U}\,\mathrm{Im}\,\mathrm{e}^{-\mathrm{i}\alpha}\Omega = \mathcal{H} + \mathcal{R}\,,$$

where $\mathcal{H}$ are harmonic functions **replacing the scalar attractor values**

- The angular momentum harmonic is the invariant $M = \langle \mathcal{H}, \mathcal{R} \rangle$
- $\mathcal{R}$ is proportional to a so-called doubly critical charge vector:

$$3|\langle \mathcal{R}, \Omega \rangle)|^2 - |D_a \langle \mathcal{R}, \Omega \rangle|^2 = 0\,,$$

- and is fixed up to normalisation by demanding:

$$I_4(\mathcal{H}, \mathcal{H}, \mathcal{H}, \mathcal{R}) = 0\,,$$

in all frames, as for the seed.

Outlook

- Our ansatz generates the full asymptotic under-rotating geometry, given the attractor solution.
- Open problem to solve the non-BPS attractor equations

Outlook

- Our ansatz generates the full asymptotic under-rotating geometry, given the attractor solution.
- Open problem to solve the non-BPS attractor equations
- The flow equations have the same form as the BPS (?!?) equations of $U(1)$ **gauged** SUGRA [Dall'Agata & Gnecchi, 2010]: doubly critical condition on $\mathcal{R}$ is identified as zero potential condition [work in progress]

Outlook

- Our ansatz generates the full asymptotic under-rotating geometry, given the attractor solution.
- Open problem to solve the non-BPS attractor equations
- The flow equations have the same form as the BPS (?!?) equations of $U(1)$ **gauged** SUGRA [Dall'Agata & Gnecchi, 2010]: doubly critical condition on $\mathcal{R}$ is identified as zero potential condition [work in progress]
- What about multi-centre solutions?

THE END

Thank you!