

Tri-Sasakian consistent reduction

Based on the homonymous arXiv: 1110.5327 with Davide Cassani

<http://itf.fys.kuleuven.be/~koerber/talks.html>

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 - Contain massive modes
 - Used to e.g. construct Lifshitz solutions

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- Tri-Sasakian foliation: S^3 or $SO(3) \rightarrow KE_4$
- η^I : dual one-forms satisfying and two-forms J^I :

$$\begin{aligned} d\eta^I &= 2J^I - \epsilon^I{}_{JK} \eta^J \wedge \eta^K \\ \iota_{\xi_I} J^J &= 0, \quad J^I \wedge J^J \propto \delta^{IJ} \end{aligned}$$

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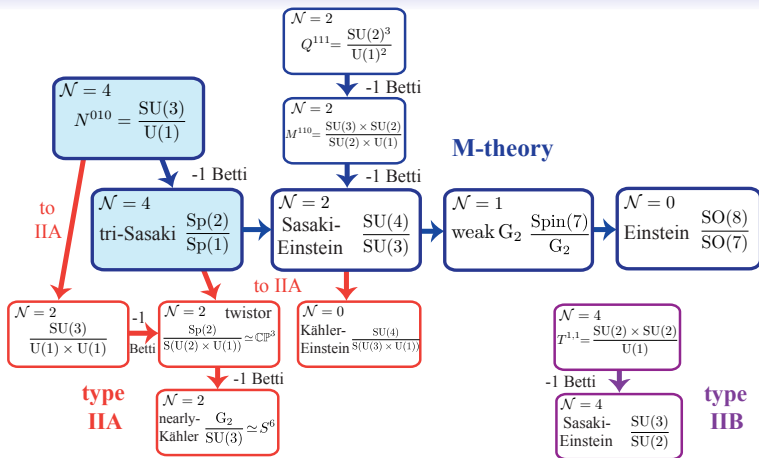
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\Rightarrow **Consistency = ensured**

- Moreover: consistency of other recent universal reductions = ensured
- However: 11D on S^7 and S^4 , IIB on S^5 remain mysterious

Overview of recent consistent truncations



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- Other solutions: $\mathcal{N} = 1$ and $\mathcal{N} = 0$ (squashed and Englert)

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