Dimensional reductions of Double Field Theory

Diego Marques

IPhT CEA Saclay

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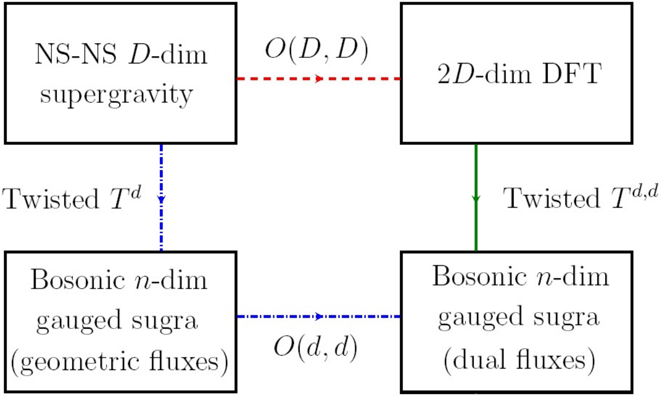
Based on Aldazabal, Baron, DM and Nuñez, 1109.0290 (JHEP)

Motivation


NS-NS D-dim supergravity
$O(D, D)$
$2D$-dim DFT
Twisted T^d
Twisted $T^{d,d}$
Bosonic n-dim gauged sugra (geometric fluxes)
$O(d, d)$
Bosonic n-dim gauged sugra (dual fluxes)

- T-duality explicit in field theory. Defined on a double space

$$X^M = (x^i, \tilde{x}_i) \qquad \leftrightarrow \qquad P_M = (p_i, w^i)$$

- Restricted DFT: can always rotate to a frame in which fields depend only on x^i

$$\partial_M \partial^M A = 0 \; , \qquad \partial_M A \partial^M B = 0$$

A, B are fields and gauge parameters

- Field content

$$\mathcal{H}_{MN}(x^i, \tilde{x}_i) \; , \qquad d(x^i, \tilde{x}_i)$$

- Generalized $2D \times 2D$ metric (constrained)

$$\mathcal{H} = \begin{pmatrix} g^{-1} & -g^{-1}b \\ bg^{-1} & g - bg^{-1}b \end{pmatrix} \in O(D, D)$$

- Invariant dilaton d

$$e^{-2d} = \sqrt{g}\, e^{-2\phi}$$

$$S_{DFT} = \int dx d\tilde{x} \; e^{-2d} \; \mathcal{R}$$

$$\mathcal{R} = \frac{1}{8}\mathcal{H}^{MN}\partial_M \mathcal{H}^{KL}\partial_N \mathcal{H}_{KL} - \frac{1}{2}\mathcal{H}^{MN}\partial_M \mathcal{H}^{KL}\partial_K \mathcal{H}_{NL} - 4\mathcal{H}^{MN}\partial_M d \partial_N d$$

When $\partial_M = (0, \partial_i)$

$$S_{DFT} \;\rightarrow\; S_{NSNS} = \int dx \sqrt{g} e^{-2\phi} \left(\mathbf{R} + 4(\partial\phi)^2 - \frac{1}{12}H^2\right)$$

Double Field Theory

Global symmetries

$$\mathcal{H}' = U^T \mathcal{H} \ U \ , \qquad U \in O(D,D)$$

Lead to Buscher rules

$$g'_{aa} = 1/g_{aa} \ , \quad g'_{ai} = -b_{ai}/g_{aa} \ , \quad g'_{ij} = g_{ij} - (g_{ai}g_{aj} - b_{ai}b_{aj})/g_{aa}$$

$$b'_{ai} = -g_{ai}/g_{aa} \ , \quad b'_{ij} = b_{ij} - (g_{ai}b_{aj} - b_{ai}g_{aj})/g_{aa}$$

Double Field Theory is invariant under generalized T-dualities.

Also invariant under gauge transformations with parameter ξ_M

$$\delta_\xi e^{-2d} = \partial_M\left(\xi^M e^{-2d}\right)$$

$$\delta_\xi \mathcal{H}^{MN} = \xi^P\partial_P\mathcal{H}^{MN} + \left(\partial^M\xi_P - \partial_P\xi^M\right)\mathcal{H}^{PN} + \left(\partial^N\xi_P - \partial_P\xi^N\right)\mathcal{H}^{MP}$$

When $\xi^M = \left(\tilde{\varepsilon}_i, \varepsilon^i\right)$ and $\partial_M = (0, \partial_i)$

$$\begin{aligned} \delta_\xi g^{ij} &= \mathcal{L}_\varepsilon g^{ij} \\ \delta_\xi b_{ij} &= \mathcal{L}_\varepsilon b_{ij} + \partial_i\tilde{\varepsilon}_j - \partial_j\tilde{\varepsilon}_i \end{aligned}$$

Double Field Theory (Hohm, Hull, Zwiebach)

Some extensions

- Heterotic formulation: Andriot; Hohm, Kwak
- Type II: Hohm, Kwak, Zwiebach; Coimbra, Strickland-Constable, Waldram
- Massive Type II: Hohm, Kwak
- U-duality: Berman, Copland, Godazgar, Perry, Thompson; West
- Generalized Geometry: Jeon, Lee, Park; Hohm, Kwak; Coimbra, Strickland-Constable, Waldram
- Non-geometry: Andriot, Larfors, Lust, Patalong
- Branes and solitons: de Boer, Shigemori; Bergshoeff, Riccioni; Albertsson, Dai, Kao, Lin; Jensen
- Effective theories: Aldazabal, Baron, DM, Nuñez; Geissbhuler

Scherk-Schwarz dimensional reductions

- Split coordinates of the spacetime $(\mathbb{X}, \mathbb{Y})$, $\mathbb{X}$: external, $\mathbb{Y}$: internal
- Reduction ansatz (not the most general)

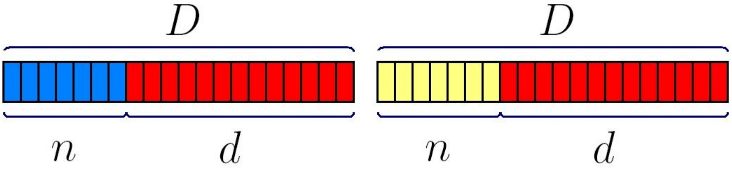
$$\mathcal{H}(\mathbb{X}, \mathbb{Y}) \to U^T(\mathbb{Y})\ \widehat{\mathcal{H}}(\mathbb{X})\ U(\mathbb{Y})\ , \qquad d(\mathbb{X}, \mathbb{Y}) \to \widehat{d}(\mathbb{X})$$

- Information of $U(\mathbb{Y})$ is encoded in the gaugings

$$f_{PQR} \equiv 3\eta_{S[P}(U^{-1})^M{}_Q(U^{-1})^N{}_{R]}\partial_M U^S{}_N$$

Effective actions of Double Field Theory

Double twisted torus



Space-time $(R \to \infty)$ $(\mathbb{X})$

~~dual Space-time~~

Double internal space $(\mathbb{Y})$

Effective actions of Double Field Theory

Effective fields

After splitting indices in external and internal

- Dilaton ϕ
- Metric $g_{\mu\nu}$
- 2-form $B_{\mu\nu}$
- $2d$ vectors $A^A{}_\mu$
- d^2 scalars $\mathcal{H}_{AB}$

Effective actions of Double Field Theory

Gauge transformations

$$\hat{\xi} = (\epsilon^\mu, \tilde{\epsilon}_\mu, \lambda^A) = (\text{diffeos, B transfs, gauge transfs})$$

$$\begin{aligned}
\delta_{\hat{\xi}} g_{\mu\nu} &= \mathcal{L}_\epsilon g_{\mu\nu} \\
\delta_{\hat{\xi}} B_{\mu\nu} &= \mathcal{L}_\epsilon B_{\mu\nu} + (\partial_\mu \tilde{\epsilon}_\nu - \partial_\nu \tilde{\epsilon}_\mu) - \left(A^A{}_\mu \partial_\nu \lambda_A - A^A{}_\nu \partial_\mu \lambda_A \right)/2 \\
\delta_{\hat{\xi}} A^A{}_\mu &= \mathcal{L}_\epsilon A^A{}_\mu - \partial_\mu \lambda^A + f^A{}_{BC}\, \lambda^B A^C{}_\mu \\
\delta_{\hat{\xi}} \mathcal{H}_{AB} &= f_{AC}{}^D\, \lambda^C \mathcal{H}_{DB} + f_{BC}{}^D\, \lambda^C \mathcal{H}_{AD}
\end{aligned}$$

Closure $\quad f^M{}_{N[P} f^N{}_{QR]} = 0$

Effective actions of Double Field Theory

Effective action

$$S_{eff} = v \int d^n x \sqrt{g} e^{-2\phi} \Big\{ \mathbf{R} + 4\, \partial^\mu \Phi \partial_\mu \Phi - \frac{1}{4} \mathcal{H}_{AB} \mathcal{F}^{A\mu\nu} \mathcal{F}^B{}_{\mu\nu}$$
$$- \frac{1}{12} \mathcal{G}_{\mu\nu\rho} \mathcal{G}^{\mu\nu\rho} + \frac{1}{8} D_\mu \mathcal{H}_{AB} D^\mu \mathcal{H}^{AB} - V \Big\}$$

$$V = \frac{1}{4} f^C{}_{DA}\, f^D{}_{CB} \mathcal{H}^{AB} + \frac{1}{12} f^E{}_{AC}\, f^F{}_{BD} \mathcal{H}^{AB}\, \mathcal{H}^{CD}\, \mathcal{H}_{EF}$$

$$\mathcal{F}^A{}_{\mu\nu} = \partial_\mu A^A{}_\nu - \partial_\nu A^A{}_\mu + f^A{}_{BC} A^B{}_\mu\, A^C{}_\nu$$

$$\mathcal{G}_{\mu\rho\lambda} = 3 \partial_{[\mu} B_{\rho\lambda]} + f_{ABC} A^A{}_\mu A^B{}_\rho A^C{}_\lambda + 3 \partial_{[\mu} A^A{}_\rho A_{\lambda]A}$$

$$D_\mu \mathcal{H}_{AB} = \partial_\mu \mathcal{H}_{AB} + f^C{}_{AD} A^D{}_\mu \mathcal{H}_{CB} + f^C{}_{BD} A^D{}_\mu \mathcal{H}_{AC}$$

- When $D = 10$ and $n = 4$ you get the electric bosonic sector of $\mathcal{N} = 4$ supergravity
- Additional vector multiplets can be added by extending DFT to include vectors
- Fundamental gaugings ξ_A are reached by extending the ansatz
- These theories are tructations of $\mathcal{N} = 8$ (when there are no additional vectors)
- Strong constraint is stronger than quadratic constraints

Backgrounds and (non)geometric fluxes

- $U(\mathbb{Y})$ can be interpreted as a generalized vielbein

$$\mathcal{H}(\mathbb{Y}) = U^T(\mathbb{Y})U(\mathbb{Y}) = \begin{pmatrix} g^{-1} & -g^{-1}b \\ bg^{-1} & g - bg^{-1}b \end{pmatrix}$$

(Dall'agata, Prezas, Samtleben, Trigiante)

- $O(d,d)$ democracy

$$f_{PQR} \equiv 3\eta_{S[P}(U^{-1})^M{}_Q(U^{-1})^N{}_{R]}\partial_M U^S{}_N$$

Geometric and non-geometric fluxes appear on an equal footing

Summary

DFT promotes a string duality to a symmetry. Gives an action for objects in generalized geometry defined on a doubled space. Scherk-Schwarz flux compactifications of DFT lead to gauged supergravities that encode gaugings asociated to non-geometric backgrounds in string theory, which from the perspective of doubled geometry are however geometric.

Summary

DFT promotes a string duality to a symmetry. Gives an action for objects in generalized geometry defined on a doubled space. Scherk-Schwarz flux compactifications of DFT lead to gauged supergravities that encode gaugings asociated to non-geometric backgrounds in string theory, which from the perspective of doubled geometry are however geometric.