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Flux compactifications and SUSY-breaking

Based on:

arXiv:0807.4540 arXiv:1004.0867 in collaboration with J. Held, P. Lüst, F. Marchesano, P. Tsimpis

15-17 November 2011, Saclay



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Provides automatically calibration structures: space-filling branes

flux

supersymmetric branes branes brane stability automatic

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Provides automatically calibration structures: flux × space-filling branes

supersymmetric branes branes brane stability automatic

Calibrated branes simplify bulk-brane coupled equations too!

Koerber & Tsimpis `07 Lüst, Marchesano, L.M. & Tsimpis `08

Figure 1 For the second-order field equations for first-order ones

Provides automatically calibration structures: flux x space-filling branes

supersymmetric branes branes brane stability automatic

Calibrated branes simplify bulk-brane coupled equations too!

Koerber & Tsimpis `07 Lüst, Marchesano, L.M. & Tsimpis `08

Can we break SUSY preserving (some of) these nice properties?

Prototypical example





$$ds_{10}^2 = e^{2A} dx^{\mu} dx_{\mu} + e^{-2A} ds_{\text{F-th}}^2$$
$$*G_3 = iG_3 \longrightarrow G_3 = F_3 + \tau H$$
$$F_5 = *de^{-4A}$$

tree-level (no-scale) SUSY-breaking

$$G^{0,3}
eq 0$$
 , $W_{ ext{tree}} = \int_X \Omega \wedge G_3
eq 0$

All bulk+branes coupled EoM's satisfied!

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All bulk+branes coupled EoM's satisfied!

What is the mechanism behind? Can we extend it to more general settings?

Strategy for general case

§ 4D approach to 10D physics







- * V as 4D potential depending on all KK-modes
- * more direct 4D interpretation on 10D equations
- * SUSY should impose a structure on V

Sinternal NS-NS fields:

metric: $ds_{10}^2 = e^{2A} dx^{\mu} dx_{\mu} + ds_X^2$ dilaton: ϕ 3-form: H = dB



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metric:
$$ds_{10}^2 = e^{2A} dx^{\mu} dx_{\mu} + ds_X^2$$

dilaton: ϕ 3-form: $H = dB$



- Solution Internal NS-NS fields: metric: $ds_{10}^2 = e^{2A} dx^{\mu} dx_{\mu} + ds_X^2$ dilaton: ϕ 3-form: H = dB
- ✤ Internal R-R fields:

$$F_{
m RR} = \sum_k F_k ~~ {k \, {
m even/odd} \over {
m in \, IIA/IIB}}$$





Full set of 10D EoM can be obtained by extremizing $F_{el} = *F_{RR}$ $V = \int_{X} e^{4A} \left\{ e^{-2\phi} \left[-R_X + \frac{1}{2}H^2 - 4(\mathrm{d}\phi)^2 + 8\nabla^2 A + 20(\mathrm{d}A)^2 \right] - \frac{1}{2}F_{el}^2 \right\}$ $+ \sum_{i \in \text{branes}} \tau_i \left(\int_{\Sigma_i} e^{4A - \phi} \sqrt{\det(g|_{\Sigma_i} + \mathcal{F}_i)} - \int_{\Sigma_i} C \wedge e^{\mathcal{F}_i} \right)$



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supersymmetry
Underlying supersymmetric
structure invisible!

- X X Space-filling branes
- Supersymmetry generators
 - $\epsilon_1 = \zeta \otimes \eta_1 + ext{ c.c.}$ $\epsilon_2 = \zeta \otimes \eta_2 + ext{ c.c.}$



Supersymmetry generators

$$\epsilon_1 = \zeta \otimes \eta_1 + \text{ c.c.}$$

$$\epsilon_2 = \zeta \otimes \eta_2 + \text{ c.c.}$$



Full information on NS-NS sector, η_1 and η_2 can be packed into two pure spinors on $T_X \oplus T_X^*$

$$\begin{aligned} \mathcal{Z} &\sim e^{3A-\phi}\eta_1 \otimes \eta_2^T \\ T &\sim e^{-\phi}\eta_1 \otimes \eta_2^\dagger \end{aligned}$$



Full information on NS-NS sector, η_1 and η_2 can be packed into two pure spinors on $T_X \oplus T_X^*$

$$\mathcal{Z} \sim e^{3A-\phi}\eta_1 \otimes \eta_2^T$$
 IIA $\mathcal{Z} = \mathcal{Z}_0 + \mathcal{Z}_2 + \mathcal{Z}_4 + \mathcal{Z}_6$
 $T \sim e^{-\phi}\eta_1 \otimes \eta_2^\dagger$ $T = T_1 + T_3 + T_5$
e.g. for $X = CY_3$: $\mathcal{Z} = e^{iJ}$, $T = e^{-\phi}\Omega^{3,0}$



Full information on NS-NS sector, η_1 and η_2 can be packed into two pure spinors on $T_X \oplus T_X^*$

$$\begin{aligned} \mathcal{Z} \sim e^{3A-\phi}\eta_1 \otimes \eta_2^T & \text{IIB} & \mathcal{Z} = \mathcal{Z}_1 + \mathcal{Z}_3 + \mathcal{Z}_5 \\ T \sim e^{-\phi}\eta_1 \otimes \eta_2^\dagger & \text{T} = T_0 + T_2 + T_4 + T_6 \end{aligned}$$
e.g. for $X = CY_3 : \mathcal{Z} = \Omega^{3,0}$, $T = e^{-\phi}e^{iJ}$

BPS form of the potential

$$V = \int_X e^{4A} \left\{ e^{-2\phi} \left[-R_X + \frac{1}{2} H^2 - 4(\mathrm{d}\phi)^2 + 8\nabla^2 A + 20(\mathrm{d}A)^2 \right] - \frac{1}{2} F_{\mathrm{el}}^2 \right\}$$
$$+ \sum_{i \in \mathrm{branes}} \tau_i \left(\int_{\Sigma_i} e^{4A - \phi} \sqrt{\det(g|_{\Sigma_i} + \mathcal{F}_i)} - \int_{\Sigma_i} C \wedge e^{\mathcal{F}_i} \right)$$

BPS form of the potential

$$V = V_{\text{branes}} - V_{\text{branes}}^{\text{BPS}}$$

$$+ \int_{X} \left[d_{H} (e^{4A} \text{Re} T) - e^{4A} * F_{\text{RR}} \right]^{2}$$

$$+ \int_{X} \left[d_{H} (e^{2A} \text{Im} T) \right]^{2}$$

$$+ \int_{X} \left| d_{H} \mathcal{Z} \right|^{2}$$

$$- \int_{X} \left| \langle T, d_{H} \mathcal{Z} \rangle \right|^{2}$$

$$- \int_{X} \left(\dots \right)^{2}$$

$$d_{H} := d + H \wedge$$

BPS form of the potential

$$V = V_{\text{branes}} - V_{\text{branes}}^{\text{BPS}} \geq 0$$

$$+ \int_{X} \left[d_{H} (e^{4A} \text{Re} T) - e^{4A} * F_{\text{RR}} \right]^{2} \geq 0$$

$$+ \int_{X} \left[d_{H} (e^{2A} \text{Im} T) \right]^{2} \geq 0$$

$$+ \int_{X} \left| d_{H} \mathcal{Z} \right|^{2} \geq 0$$

$$- \int_{X} \left| \langle T, d_{H} \mathcal{Z} \rangle \right|^{2} \leq 0$$

$$- \int_{X} (\dots)^{2} \leq 0$$

$$d_{H} := d + H \wedge$$

BPS form of the potential Koerber & L.M. `07 **4D INTERPRETATION** $V = V_{\rm branes} - V_{\rm branes}^{\rm BPS}$ > 0(brane BPS-bound) $+ \int_{\mathbf{V}} \left[\mathrm{d}_H(e^{4A} \mathrm{Re}\,T) - e^{4A} * F_{\mathrm{RR}} \right]^2$ $\sim |F_{\mathcal{Z}}|^2$ > 0 $\sim |\mathcal{D}|^2$ $+\int_{\mathbf{V}} \left[\mathrm{d}_{H}(e^{2A}\mathrm{Im}\,T) \right]^{2}$ > 0 $+\int_{\mathbf{V}} \left| \mathrm{d}_{H} \mathcal{Z} \right|^{2}$ $\sim |F_T|^2$ > 0 $-\int_{X} |\langle T, d_{H} \mathcal{Z} \rangle|^{2}$ $-\int_{X} (\ldots)^{2}$ $\sim |F_T|^2$ < 0 \sim $|\mathcal{D} + F_T|^2$ < 0 $d_H := d + H \wedge$

$$V = V_{\text{branes}} - V_{\text{branes}}^{\text{BPS}}$$
$$+ \int_{X} \left[d_{H} (e^{4A} \text{Re} T) - e^{4A} * F_{\text{RR}} \right]^{2}$$
$$+ \int_{X} \left[d_{H} (e^{2A} \text{Im} T) \right]^{2}$$
$$+ \int_{X} \left| d_{H} \mathcal{Z} \right|^{2}$$
$$- \int_{X} \left| \langle T, d_{H} \mathcal{Z} \rangle \right|^{2}$$
$$- \int_{X} \left(\dots \right)^{2}$$

$$V = V_{\text{branes}} - V_{\text{branes}}^{\text{BPS}}$$

$$+ \int_{X} \left[d_{H} (e^{4A} \text{Re} T) - e^{4A} * F_{\text{RR}} \right]^{2} = 0$$

$$+ \int_{X} \left[d_{H} (e^{2A} \text{Im} T) \right]^{2} = 0$$

$$+ \int_{X} \left| d_{H} \mathcal{Z} \right|^{2} = 0$$

$$- \int_{X} \left| \langle T, d_{H} \mathcal{Z} \rangle \right|^{2} = 0$$

$$- \int_{X} \left(\dots \right)^{2} = 0$$

$$d_H(e^{4A} \operatorname{Re} T) = e^{4A} * F_{\mathrm{RR}}$$

*
$$d_H(e^{2A} \operatorname{Im} T) = 0$$

*
$$\mathrm{d}_H \mathcal{Z} = 0$$

Simplest solution of EoM's

$$V = V_{\text{branes}} - V_{\text{branes}}^{\text{BPS}} + \int_X \left[d_H (e^{4A} \text{Re} T) - e^{4A} * F_{\text{RR}} \right]^2 = 0 \qquad * d_H \\ + \int_X \left[d_H (e^{2A} \text{Im} T) \right]^2 = 0 \\ + \int_X |d_H \mathcal{Z}|^2 = 0 \\ - \int_X |\langle T, d_H \mathcal{Z} \rangle|^2 = 0 \\ - \int_X (\dots)^2 = 0$$

calibration for space-filling branes * $d_H(e^{4A} \operatorname{Re} T) = e^{4A} * F_{\mathrm{RR}}$ * $d_H(e^{2A} \operatorname{Im} T) = 0$

*
$$d_H \mathcal{Z} = 0$$

calibration for

$$V = V_{\text{branes}} - V_{\text{branes}}^{\text{BPS}}$$

$$+ \int_{X} \left[d_{H}(e^{4A}\text{Re}T) - e^{4A} * F_{\text{RR}} \right]^{2} = 0$$

$$+ \int_{X} \left[d_{H}(e^{2A}\text{Im}T) \right]^{2} = 0$$

$$+ \int_{X} \left[d_{H}(e^{2A}\text{Im}T) \right]^{2} = 0$$

$$+ \int_{X} \left| d_{H}Z \right|^{2} = 0$$

$$- \int_{X} \left| \langle T, d_{H}Z \rangle \right|^{2} = 0$$

$$- \int_{X} (\dots)^{2} = 0$$

$$\frac{\text{space-filling branes}}{\text{space-filling branes}}$$

$$V = V_{\text{branes}} - V_{\text{branes}}^{\text{BPS}} = 0$$

+ $\int_X \left[d_H (e^{4A} \text{Re} T) - e^{4A} * F_{\text{RR}} \right]^2 = 0$
+ $\int_X \left[d_H (e^{2A} \text{Im} T) \right]^2 = 0$
+ $\int_X \left| d_H \mathcal{Z} \right|^2 = 0$
- $\int_X \left| \langle T, d_H \mathcal{Z} \rangle \right|^2 = 0$
= 0
- $\int_X \left| \langle T, d_H \mathcal{Z} \rangle \right|^2 = 0$

calibration for
space-filling branes
*
$$d_H(e^{4A} \operatorname{Re} T) = e^{4A} * F_{RR}$$

* $d_H(e^{2A} \operatorname{Im} T) = 0$
* $d_H \mathcal{Z} = 0$
plus calibrated space-
filling branes

$$V = V_{\text{branes}} - V_{\text{branes}}^{\text{BPS}} = 0$$

$$+ \int_{X} \left[d_{H}(e^{4A}\text{Re}T) - e^{4A} * F_{\text{RR}} \right]^{2} = 0$$

$$+ \int_{X} \left[d_{H}(e^{2A}\text{Im}T) \right]^{2} = 0$$

$$+ \int_{X} \left[d_{H}(e^{2A}\text{Im}T) \right]^{2} = 0$$

$$+ \int_{X} \left| d_{H}Z \right|^{2} = 0$$

$$- \int_{X} \left| \langle T, d_{H}Z \rangle \right|^{2} = 0$$

$$- \int_{X} \left(\dots \right)^{2} = 0$$

$$+ \int_{X} \left| (T, d_{H}Z) \right|^{2} = 0$$

$$= 0$$

$$Calibrated space-filling branes = 0$$

$$Calibrated space-filling branes =$$

Simplest solution of EoM's

$$V = V_{\text{branes}} - V_{\text{branes}}^{\text{BPS}} = 0$$

$$+ \int_{X} \left[d_{H}(e^{4A}\text{Re}T) - e^{4A} * F_{\text{RR}} \right]^{2} = 0$$

$$+ \int_{X} \left[d_{H}(e^{2A}\text{Im}T) \right]^{2} = 0$$

$$+ \int_{X} \left[d_{H}Z \right]^{2} = 0$$

$$- \int_{X} \left| \langle T, d_{H}Z \rangle \right|^{2} = 0$$

$$- \int_{X} \left(\dots \right)^{2} = 0$$

$$+ \int_{X} \left(\dots \right)^{2} = 0$$

manifestly satisfied!

conditions!

Petrini & Tomasiello `05

calibration for

Solution SUSY-breaking ansatz

$$V = V_{\text{branes}} - V_{\text{branes}}^{\text{BPS}}$$
$$+ \int_{X} \left[d_{H} (e^{4A} \text{Re} T) - e^{4A} * F_{\text{RR}} \right]^{2}$$
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$$+ \int_{X} \left| d_{H} \mathcal{Z} \right|^{2}$$
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$$- \int_{X} \left(\dots \right)^{2}$$

Solution SUSY-breaking ansatz

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$$+ \int_{X} \left| d_{H}Z \right|^{2}$$

$$- \int_{X} \left| \langle T, d_{H}Z \rangle \right|^{2} = 0$$

calibration for

Solution SUSY-breaking ansatz

$$V = V_{\text{branes}} - V_{\text{branes}}^{\text{BPS}}$$

$$+ \int_{X} \left[d_{H}(e^{4A}\text{Re}T) - e^{4A} * F_{\text{RR}} \right]^{2} = 0$$

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$$+ \int_{X} \left[d_{H}(e^{2A}\text{Im}T) \right]^{2} = 0$$

$$+ \int_{X} \left| d_{H}\mathcal{Z} \right|^{2}$$

$$- \int_{X} \left| \langle T, d_{H}\mathcal{Z} \rangle \right|^{2} = 0$$

Solution SUSY-breaking ansatz

$$V = V_{\text{branes}} - V_{\text{branes}}^{\text{BPS}}$$

$$+ \int_{X} \left[d_{H}(e^{4A} \operatorname{Re} T) - e^{4A} * F_{\text{RR}} \right]^{2} = 0$$

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$$+ \int_{X} \left| d_{H} \mathcal{Z} \right|^{2}$$

$$- \int_{X} \left| \langle T, d_{H} \mathcal{Z} \rangle \right|^{2} = 0$$

calibration for space-filling branes

*
$$d_H(e^{4A} \operatorname{Re} T) = e^{4A} * F_{\mathrm{RR}}$$

* $d_H(e^{2A} \operatorname{Im} T) = 0$

plus calibrated branes

= 0

= 0

= 0

= 0

Solution SUSY-breaking ansatz

 $V = V_{\rm branes} - V_{\rm branes}^{\rm BPS}$

calibration for space-filling branes $d = (e^{4A} P e^{T}) = e^{4A} + F$

plus calibrated branes

$$+ \int_{X} \left[d_{H}(e^{2A} \operatorname{Im} T) \right]^{2}$$
$$+ \int_{X} \left| d_{H} \mathcal{Z} \right|^{2}$$
$$- \int_{X} \left| \langle T, d_{H} \mathcal{Z} \rangle \right|^{2}$$
$$- \int_{X} (\ldots)^{2}$$

 $+ \int_{Y} \left[\mathrm{d}_{H}(e^{4A} \mathrm{Re}\,T) - e^{4A} * F_{\mathrm{RR}} \right]^{2}$

Solution Natura SUSY-breaking ansatz



compactification to $Mink_4$

Fake fibration with calibrated fibers

 $\Pi \hookrightarrow X \to \mathcal{B}$



Solution Take fibration with calibrated fibers $\Pi \hookrightarrow X \to \mathcal{B}$

Section Choose SUSY-breaking of the form

 $d_H \mathcal{Z} = r e^{-R} \operatorname{vol}_{\mathcal{B}}$ SUSY-breaking twisting

ISY-breaking twisting parameter two-form $\mathrm{d}R=H|_{\Pi}$



Take fibration with calibrated fibers $\Pi \hookrightarrow X \to \mathcal{B}$

Choose SUSY-breaking of the form

 $d_H \mathcal{Z} = r \, e^{-R} \operatorname{vol}_{\mathcal{B}}$

SUSY-breaking

twisting parameter two-form $dR = H|_{\Pi}$



In the wCY case: {fibers Π } = {points in X} $\mathcal{B} = X$ $\mathcal{Z} = \Omega_{\mathrm{CY}} \rightarrow \mathrm{d}_H \mathcal{Z} = \mathrm{d}_H \Omega_{\mathrm{CY}} = r \operatorname{vol}_X$ $d\Omega_{
m CY} = 0$, $r \simeq H^{0,3}$

Solution Take fibration with calibrated fibers $\Pi \hookrightarrow X \to \mathcal{B}$

Section Choose SUSY-breaking of the form

 $d_H \mathcal{Z} = r \, e^{-R} \operatorname{vol}_{\mathcal{B}}$

SUSY-breaking parameter

twisting two-form $\mathrm{d}R=H|_{\Pi}$



Explicit examples on twisted tori

see also Camara & Graña `08

Blaback, Danielsson, Junghans, Van Riet, Wrase & Zagermann `10



Open problems

Existence theorems? as in SUSY-case

Higher order corrections?
Combination with quantum effects?



Extension of the strategy to de Sitter?

Andriot, Goi, Minasian & Petrini `08



CALIBRATIONS for D-branes



CALIBRATIONS for D-branes



The problem

Find non-supersymmetric flux compactifications

Seak supersymmetry in a controlled way

In particular, keep back-reaction of localized sourced (Dbranes and orientifold) under control