Microstate geometries from string amplitudes

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Outline of the talk

- The idea
  - To derive the precise supergravity solution of a (black hole) microstate we need detailed information about the source
  - Disk amplitudes describe how closed strings interact with D-brane bound states and provide (in principle) all relevant information
- The results (so far)
  - $1/4$-BPS systems are under control and provide the perfect arena to learn how to connect string and supergravity calculations
  - $1/8$-BPS systems are the most interesting ones. . . we analyzed a class of 3-charge microstate and found a few surprises already!
- The main open problem
  - For each microstate connect the large r (flat space) and the small r (AdS) limits

The idea

- The usual approach in supergravity: write down the spinor variations and the e.o.m. and solve them thanks to a smart ansatz
- All info about the source are encoded in the ansatz
- Example: take 1/2-BPS Dp-brane solutions. The ansatz preserves $SO(1,p) \times SO(9-p)$

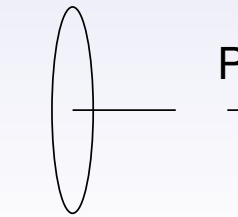
$$ds^2 = (H(r))^{\alpha} \left(\eta_{\alpha\beta} dx^{\alpha} dx^{\beta}\right) + (H(r))^{\beta} \left(\delta_{ij} dx^i dx^j\right) .$$

Then $\alpha = \frac{p-7}{8}$, $\beta = \frac{p+1}{8}$ and $H(r)$ is a harmonic function

$$H(r) = 1 + \left(\frac{R_p}{r}\right)^{7-p}, \quad R_p^{7-p} = \frac{gN(2\pi\sqrt{\alpha'})^{7-p}}{(7-p)\Omega_{8-p}} , \quad \Omega_n = \frac{2\pi^{\frac{n+1}{2}}}{\Gamma(\frac{n+1}{2})}.$$

- Expand it for $r \gg R_p$: each term in this expansion correspond to a supergravity Feynamn diagram with a source

- At the linear level the fields satisfy a harmonic equation in the transverse space $\nabla^2\phi = \delta\Gamma^{\text{bound}}/\delta\phi$ where Γ^{bound} is the DBI action.
- Thus we have $\phi = \nabla^{-2}\left(\delta\Gamma^{\text{bound}}/\delta\phi\right)$, which diagrammatically is



$$= g_{ii}^{\text{as}}(k_i)\,, \qquad g_{ii}^{\text{as}}(x) = \int g_{ii}^{\text{as}}(k)e^{ikx}\frac{d^{9-p}k}{(2\pi)^{9-p}}$$

$$g_{ii}(x) \sim \eta_{ii} + g_{ii}^{\text{as}}(x) + \mathcal{O}\left((R_p/r)^{2(7-p)}\right)$$

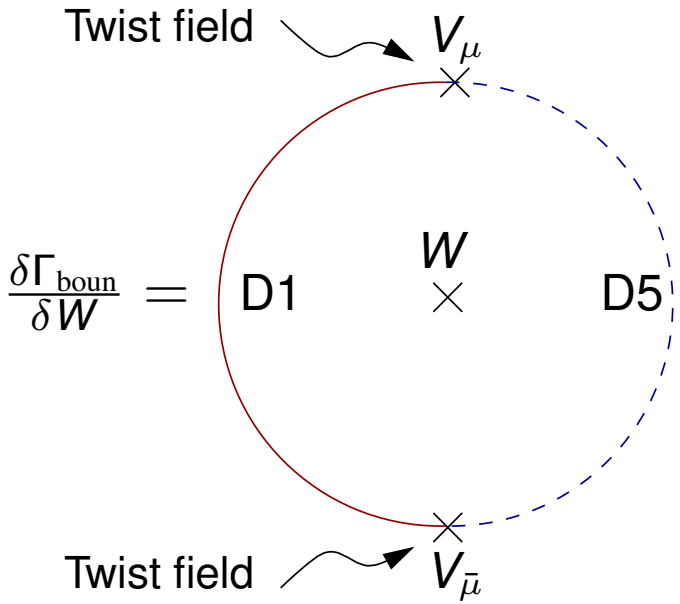
- Higher order terms correspond to diagrams with more sources, which describe the perturbative solution of $\nabla^2\phi + \sigma\phi^2 = \delta\Gamma^{\text{bound}}/\delta\phi$
- 1/4-BPS and 1/8 microstates are described by the same diagrams, but with more complicated sources (not just DBI!)
- Disk amplitudes with a single closed string state give precisely the source $\delta\Gamma^{\text{bound}}/\delta\phi$

The results

- Let us consider type IIB compactified on $R_t \times S^1 \times R^4 \times T^4$
- For a 1/4-BPS microstate sourced by D1 and D5 brane the string frame metric is

$$ds^2 = \frac{-(dt-A)^2+(dy+B)^2}{(H_{1,f}H_{5,f})^{\frac{1}{2}}} + (H_{1,f}H_{5,f})^{\frac{1}{2}} dx_i^2 + \sqrt{\frac{H_{1,f}}{H_{5,f}}} dx_a^2 \,,$$

  where $A = A_i dx_i$ and $dB = - *_4 \, dA$.
- The new contributions in the large r expansion are captured by

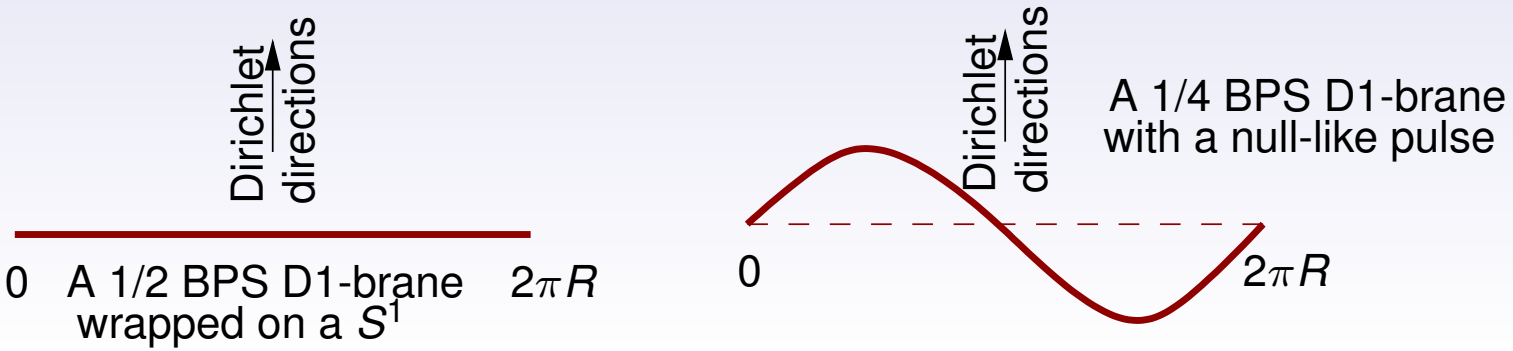


$$V_\mu = \mu^A e^{-\frac{\varphi}{2}} S_A \, \Delta \,, \quad V_{\bar{\mu}} = \bar{\mu}^A e^{-\frac{\varphi}{2}} S_A \, \bar{\Delta} \,,$$

S_A is a Weyl spinor of $SO(1,5)$ acting on t, y, x_i

W is a massless closed string state

- We can use the same idea, but with oscillating D1 and D5 branes. If the wave is purely left (or right) moving the system preserves only $1/2$ of the original susys



- The reflection matrix depends on $f_i(v)$ $(v = t + y)$ determining the D1's position in the Dirichlet directions

$$(R)^{\mu}{}_{\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 4|\dot{f}(V)|^2 & 1 & -4\dot{f}_i(V) & 0 \\ 2\dot{f}_i(V) & 0 & -\mathbb{1} & 0 \\ 0 & 0 & 0 & -\mathbb{1} \end{pmatrix},$$

where the indices are ordered $(v, u = t - y, i)$

- We find that all 10D IIB massless fields are non-trivial at $\mathcal{O}(1/r^4)$
- As expected, the solution contains many dipols
- The dipoles are determined by the open string data (the $\bar{\mu}\mu$ condensate and $f_i(v)$)
- The 4d metric derived from $\mathcal{A}_{\rm gra}^{\rm D1-D5}$ is conformally hyperkhaler (at least perturbatively), but not conformally flat
- We checked that it is possible to find a full non-linear 1/8-BPS ansatz compatible with the string results
- The ansatz is written à la Bena-Warner but in terms of four vector superfields
- The construction seems to yield a particular class of superstrata

The problem

- Clearly we would like to find a (set of) particular solutions within the general ansatz that
  - As $r \to \infty$ behave as dictated by the string amplitudes
  - For small r reduce to a deformation of $AdS_3 \times S^3 \times T^4$
  - Have a singularity structure that can be interpreted within string theory
- This seems to be rather difficult
- There is no single 3-charge case where these three questions are under control at the same time (this is in contrast with the 2-charge cases)
- Work is in progress ...