The physics of non-BPS black holes

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Outline

- Motivation & goals
  1. Remarks on fuzzballs
  2. Thermalisation/Scrambling vs fuzzball
- Extremal $J = 0$ non-BPS
  1. Solution & charges
  2. Different frames provide different insights/checks
  3. Constituent model
  4. Checks : probe and sugra multi-center solutions
- Extremal $J \neq 0$ non-BPS (see talk by Yeranyan)
  1. Almost BPS
  2. Composite non-BPS (see talk by Bossard)
- Non-extremal (see talks by Vercnocke, Puhm, Denef)

From BPS to non-BPS : motivation & challenges

Finite temperature BHs are non-extremal $\Longrightarrow$ Non-BPS BHs

Open problems

- Gravitational collapse, BH formation, critical behaviour, ...
- Information paradox
- Thermalisation $\Longleftrightarrow$ foundations of (non-equilibrium) statistical mechanics
- Emergence of spacetime, classical causal structure

Immediate goals & tools

- Entropy counting
- BH deconstruction, constituent models, fuzzball, ...
- Probes

Entropy counting

1. Identification of the relevant degrees of freedom
   (in the regime of parameters where BHs exist !!)
2. Counting degeneracy of states

Challenges

- Overcome susy non-renormalisation theorems (when using 'open string' description)
- Develop strongly coupled tools dealing with non-planarity

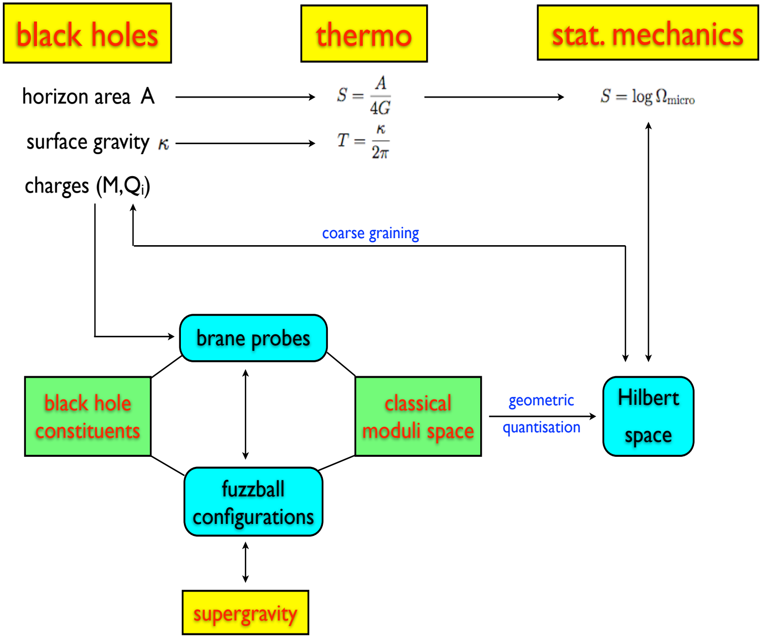
Hopes

- Semiclassical methods : extending Brown-Henneaux $\Longrightarrow$ Kerr/CFT, hidden symmetries, ...
- Integrability ?, special kinematic regimes in observables, ...
- Use of typicality : probability theory & laws of large numbers constraining the universal behaviour given some properties of the hamiltonian system

BH deconstruction, constituent models & fuzzball

- Branes expand/polarise in the presence of fluxes
  1. Supertubes in flat space $\Longleftrightarrow$ (small) D1-D5 system
  2. Giant gravitons in AdS $\Longleftrightarrow$ LLM geometries

  These mechanisms also exist in non-BPS set-ups
- BH charges split into fundamental bits/constituents
  Entropy from geometric quantisation of the classical moduli space of such configurations
- Existence of multi-center configurations : with/out horizons, regular, scaling solutions, ...


black holes
thermo
stat. mechanics
horizon area A
$S = \frac{A}{4G}$
$S = \log \Omega_{\rm micro}$
surface gravity κ
$T = \frac{\kappa}{2\pi}$
charges (M,Q_i)
coarse graining
brane probes
black hole constituents
classical moduli space
geometric quantisation
Hilbert space
fuzzball configurations
supergravity

Probes

1. Supertubes and giant gravitons were first discovered using this approximation
2. Single (abelian) branes effective actions known in any curved on-shell background
3. Well defined regime of validity

Some caveats

- Do some of these calculations capture the full dynamics ? Complicated bound states may have intrincate interactions among its constituents
- Non-abelian effects may be required
    1. Turn-on wold volume fluxes in high dimensional branes inducing further charges
    2. Consider non-abelian D-branes in curved backgrounds (not fully understood)

Comments on Fuzzball

Main conceptual idea

Quantum gravity effects not merely confined to the singularity, but spread over larger scales (horizon scale)

Large phase space is responsible for this [Mathur]

1. Consistent with holographic principle
2. Can resolve the information paradox

Comment based on AdS/CFT

BHs $\sim$ thermal states (canonical ensemble)

Lorentzian BHs do have singularities

Euclidean BHs are smooth (with suitable b.c. !!), but have NO information about the interior of the BH, only about the degeneracy of states (saddle point)

Rules for semiclassical lorentzian/euclidean gravitational path integrals ?

Thermalisation : Bulk perspective

- Causality & no-cloning considerations $\implies \tau \sim \beta \log S$ (scrambling time) [Hayden-Preskill; Sekino-Susskind;Susskind] Much faster than any diffusion process !!
- Whatever picture you believe in $\implies \exists$ some d.o.f on the (stretched) horizon : how can you explain this time scale ?

Thermalisation : Field theory perspective

- How do we describe the approach to thermal equilibrium in QM ? Are notions such as 'quantum ergodicity' meaningful ? [von Neumann;Srenidcki]
- Spectrum of heavy non-susy excitations may be very complicated ... use a distribution of level spacings (as in nuclear physics !!) : can we identify generic features giving rise to such fast time scales ?
- Can we think of the bulk fuzzball picture in terms of such level spacing distributions and declare that a single quanta thrown to the black hole will have access, through interactions, to a non-trivial phase space making fast scrambling possible ?

A non-extremal fuzzball example & non-trivial check

Facts

1. First solutions of non-extremal fuzzball configurations were found by Jejjala, Madden, Ross & Titchener in the D1-D5-P system
2. They were found to be unstable by Cardoso, Dias, Hovdebo,Myers

Feature or bug ?

Even though these configurations are *atypical* and *unstable*, it was proved by Chowdhury & Mathur that the energy radiated through these unstable modes $\equiv$ Hawking radiation for these modes

Even if we may not agree on whether supergravity (through proper geometric quantisation) should contain *all* the entropy of the black hole, these calculations do give evidence for
Hawking radiation $\sim$ leakage of energy from a complicated 'fuzzball cap'

Non-BPS supergravity solutions

Asymptotically flat solutions in STU, N=2 set-ups coming from d=11 or type IIA/B supergravity

1. Extremal non-BPS
   - J=0 : **general** single center and (partial) multi-center solution
     + constituent model
   - $J \neq 0$: **general** single center
     1. Almost BPS multi-center (single orientation change)
     2. Composite non-BPS multi-center (two orientation changes)
2. Non-extremal non-BPS (partial results)

Solutions obtained via

- U-duality in d=4
- M-theory ansatz
- Coset construction & nilpotent orbits (group theory methods)

Dictionary between them well understood

Extremal J=0 general solution

Theory : **STU** truncation (IIA on T^6)physical fields

- Physical fields : 4d metric, 4 gauge fields A^Λ,
  3 complex scalars $z^j = x^j - iy^j$
- Charges : M, J, $\Gamma = (P^\Lambda,\, Q_\Sigma)$
  $P^0 \equiv$ D6, $P^i \equiv$ D4, $Q_0 \equiv$ D0, $Q_i \equiv$ D2-brane

Parameter counting

- Full solution has 16 parameters :

$$M,\, J,\, \left(Q_\Lambda,\, P^\Lambda\right),\, z^i_\infty \qquad \Lambda = (0, j) \quad j = 1, 2, 3$$

- Absence of angular momentum J=0
- Extremality $\Longrightarrow$ M=M(Q_Λ, P^Λ, z^i_∞)
- U-duality : $\exists\ \mathrm{SL}(2, \mathbb{R})^3 \quad \Longrightarrow$ 9 parameters

$\overline{\text{D0}}$-D4-D4-D4 frame

- Standard static spherically symmetric **metric**

$$ds^2 = -e^{2U}\, dt^2 + e^{-2U}\left(dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)\right),$$

where

$$e^{-4U} = -4H_0 H^1 H^2 H^3 - B^2,$$

is determined by four harmonic functions

$$\sqrt{2}H_0 = -(1 + B^2) + \frac{\sqrt{2}Q_0}{r} \quad , \; \sqrt{2}H^i = 1 + \frac{\sqrt{2}P^i}{r}.$$

- **Gauge** fields & **Scalar** moduli

$$z^i = \frac{B - i\, e^{-2U}}{s_{ijk} H^j H^k}.$$

- 5 parameters : Q_0, P^i and 3-equal B field.

[Cardoso,Ceresole,Dall'Agata,Oberreuter,Perz][Gimon,Larsen,S]

Mass

- Non-BPS mass ($Q_0 < 0$)

$$2G_N\, M_{\text{Non-BPS}} = \frac{1}{\sqrt{2}} \left(|Q_0| + \sum_{i=1}^{3} P^i \,(1 + B^2) \right)$$

Sum of 4 half-BPS constituent masses with no binding energy

- BPS mass ($Q_0 > 0$)

$$2G_N\, M_{\text{BPS}} = 2\,|Z_\Gamma| = \frac{1}{\sqrt{2}} \left| Q_0 + \sum_{i=1}^{3} P^i \,(1 + iB)^2 \right|$$

Existence of non-trivial binding energy

- Mass difference

$$8G_N^2\,(M^2_{\text{Non-BPS}} - M^2_{\text{BPS}}) = 4\big|Q_0\big|\, B^2 \sum_i P^i \geq 0$$

only vanishes for $B = 0$.

D0-D6 frame

U-duality transformation

- Use U-duality transformations

$$M_j = \begin{pmatrix} a_j & b_j \\ c_j & d_j \end{pmatrix} : \quad z^j \longrightarrow \frac{a_j\, z^j + b_j}{c_j\, z^j + d_j}$$

to map a D0-D6 (q_0, p^0) system with a $\overline{\text{D0}}$-D4-D4-D4 $(q<0, p^i)$ system using the $\mathrm{SL}(2,\mathbb{R})$ transformation properties of the 3-tensor

$$p^0 = a_{111},\; p^1 = a_{011},\; p^2 = a_{101},\; p^3 = a_{110}$$

$$q_0 = -a_{000},\; q_1 = a_{100},\; q_2 = a_{010},\; q_3 = a_{001}$$

$$a'_{i'j'k'} = (M_1)_{i'}{}^{i}\,(M_2)_{j'}{}^{j}\,(M_3)_{k'}{}^{k}\; a_{ijk}$$

- Solution

$$M_i = \frac{-1}{\sqrt{2\lambda\,\rho_i}} \begin{pmatrix} \rho_i\lambda & -\rho_i \\ \lambda & 1 \end{pmatrix}, \quad \rho_i = \sqrt{\frac{-qp^i}{\frac{1}{2} s_{ijk}\, p^j p^k}}, \quad \lambda = \left(\frac{p^0}{q_0}\right)^{1/3}$$

D0-D6 frame

Parameters & Mass

- 5 Parameters : charges Q_0, P^0 and 3 B-fields B_i determined in terms of $\Lambda_i = \lambda\, v_i$

$$\Lambda_1\,\Lambda_2\,\Lambda_3 = \frac{P^0}{Q_0}$$
$$\Lambda_1\,(1+B_1^2) - \Lambda_1^{-1} = \Lambda_2\,(1+B_2^2) - \Lambda_2^{-1} = \Lambda_3\,(1+B_3^2) - \Lambda_3^{-1}$$

- Mass

$$\begin{aligned}
2^{3/2}\, G_N\, M &= \frac{P^0}{4}\Big(1+(\Lambda_1^{-1}+B_1)^2\Big)^{1/2}\Big(1+(\Lambda_2^{-1}+B_2)^2\Big)^{1/2}\Big(1+(\Lambda_3^{-1}+B_3)^2\Big)^{1/2} \\
&+ \frac{P^0}{4}\Big(1+(\Lambda_1^{-1}+B_1)^2\Big)^{1/2}\Big(1+(\Lambda_2^{-1}-B_2)^2\Big)^{1/2}\Big(1+(\Lambda_3^{-1}-B_3)^2\Big)^{1/2} \\
&+ \frac{P^0}{4}\Big(1+(\Lambda_1^{-1}-B_1)^2\Big)^{1/2}\Big(1+(\Lambda_2^{-1}+B_2)^2\Big)^{1/2}\Big(1+(\Lambda_3^{-1}-B_3)^2\Big)^{1/2} \\
&+ \frac{P^0}{4}\Big(1+(\Lambda_1^{-1}-B_1)^2\Big)^{1/2}\Big(1+(\Lambda_2^{-1}-B_2)^2\Big)^{1/2}\Big(1+(\Lambda_3^{-1}+B_3)^2\Big)^{1/2}
\end{aligned}$$

D0-D6 : constituent interpretation

- Transformed charge vectors : D6-branes with fluxes

$$\begin{aligned}
\Gamma_I &= \frac{1}{4}\Big(P^0; -P^0/\Lambda_1, -P^0/\Lambda_2, -P^0/\Lambda_3; Q_0; P^0/(\Lambda_2\Lambda_3), P^0/(\Lambda_1\Lambda_3), P^0/(\Lambda_1\Lambda_2)\Big) \\
\Gamma_{II} &= \frac{1}{4}\Big(P^0; -P^0/\Lambda_1, P^0/\Lambda_2, P^0/\Lambda_3; Q_0; P^0/(\Lambda_2\Lambda_3), -P^0/(\Lambda_1\Lambda_3), -P^0/(\Lambda_1\Lambda_2)\Big) \\
\Gamma_{III} &= \frac{1}{4}\Big(P^0; P^0/\Lambda_1, -P^0/\Lambda_2, P^0/\Lambda_3; Q_0; -P^0/(\Lambda_2\Lambda_3), P^0/(\Lambda_1\Lambda_3), -P^0/(\Lambda_1\Lambda_2)\Big) \\
\Gamma_{IV} &= \frac{1}{4}\Big(P^0; P^0/\Lambda_1, P^0/\Lambda_2, -P^0/\Lambda_3; Q_0; -P^0/(\Lambda_2\Lambda_3), -P^0/(\Lambda_1\Lambda_3), P^0/(\Lambda_1\Lambda_2)\Big)
\end{aligned}$$

- Total charge vector & mass

$$\begin{aligned}
\Gamma &= \Gamma_I + \Gamma_{II} + \Gamma_{III} + \Gamma_{IV} = (P^0; \vec{0}; Q_0; \vec{0}) \\
M &= M_I + M_{II} + M_{III} + M_{IV}
\end{aligned}$$

D0-D6 : Open string description

Matching Taylor's picture-I

- d=1+6 non-abelian DBI action with gauge group $\mathrm{U}(4N)$ and gauge field fluxes

$$(F_{12},\, F_{34},\, F_{56})^{I} = (f_1,\, f_2,\, f_3)$$
$$(F_{12},\, F_{34},\, F_{56})^{II} = (f_1,\, -f_2,\, -f_3)$$
$$(F_{12},\, F_{34},\, F_{56})^{III} = (-f_1,\, f_2,\, -f_3)$$
$$(F_{12},\, F_{34},\, F_{56})^{IV} = (-f_1,\, -\, f_2,\, f_3)$$

induces zero D2 and D4-brane charges, but non-trivial D0-brane charge

$$n_0 = -\frac{n_6}{4}\frac{1}{6(2\pi)^3}\int \mathrm{tr} F\wedge F\wedge F = -\frac{n_6\, V_6\, f_1\, f_2\, f_3}{(2\pi)^3}$$

D0-D6 : Open string description

Matching Taylor's picture-II

- Matching the sugra charge constraints the worldvolume fluxes

$$\frac{P^0}{Q_0} = \frac{M_6}{M_0}\frac{n_6}{n_0} = -\frac{1}{(2\pi\alpha')^3 f_1 f_2 f_3}$$

- Matching gauge fluxes f_i with background parameters Λ_i

$$\Lambda_i = -\frac{1}{2\pi\alpha' f_i}$$

- DBI mass equals supergravity ADM mass

$$M_{\mathrm{DBI}} = T_6 \int \mathrm{Tr}\sqrt{\det\left(G + (2\pi\alpha' F - B)\right)} = M_{\text{Non-BPS}}$$

D0-D6: Supergravity vs non-abelian DBI

- Masses agree (why ?)
- Non-abelian configurations exist for any value of the gauge fluxes f_i
- Supergravity requires non-linear relation between fluxes & B-fields
- Stability ??

Intersecting D3-brane frame

- Constituents : D3-branes making an angle ϕ_i with the i-th T^2

$$\cot\phi_i = 2\pi\alpha' f_i - B_i = -(\Lambda_i^{-1} + B_i)$$

- Relative angle between pairs of D3-branes :

$$\cot\vartheta_i^{AB} = \cot(\phi_i^A - \phi_i^B) = -\frac{1}{2}\left[\Lambda_i(1+B_i^2) - \Lambda_i^{-1}\right]$$

Intersecting D3-brane spectrum

Boundary conditions on the open strings stretched between each pair of D3-branes are twisted due to the relative angles ϑ's. The lightest states are 4 complex scalar fields in the NS sector, living at the intersection, with masses

$$\alpha' m_1^2 = \frac{1}{2\pi}\left(-\vartheta_1 + \vartheta_2 + \vartheta_3\right),$$
$$\alpha' m_2^2 = \frac{1}{2\pi}\left(\vartheta_1 - \vartheta_2 + \vartheta_3\right),$$
$$\alpha' m_3^2 = \frac{1}{2\pi}\left(\vartheta_1 + \vartheta_2 - \vartheta_3\right),$$
$$\alpha' m_4^2 = 1 - \frac{1}{2\pi}\left(\vartheta_1 + \vartheta_2 + \vartheta_3\right)$$

Intersecting D3-brane stability-I

Absence of perturbative tachyons

A pair of constituent $D3$-branes is perturbatively stable if their relative angles satisfy

$$\vartheta_2 + \vartheta_3 \geq \vartheta_1, \qquad \vartheta_1 + \vartheta_3 \geq \vartheta_2$$
$$\vartheta_1 + \vartheta_2 \geq \vartheta_3, \qquad 2\pi \geq \vartheta_1 + \vartheta_2 + \vartheta_3$$

Our system

For any pair (A,B), there always exists one T^2 whose relative angle vanishes

$$\exists\, j \text{ such that } \vartheta_j^{AB} = 0$$

The other two relative angles are equal

$$\vartheta_i^{AB} = \vartheta_k^{AB} \quad i \neq k \ (i, k \neq j) \quad \forall A, B$$

Stability & Susy breaking

Conclusion

- Supergravity fluxes are perturbatively stable
- There are more general solutions to the perturbative conditions

Supersymmetry

- Each pair of constituents preserves susy
- Each triple of constituents preserves susy
- Susy is only broken when we add the fourth constituent
- Are there instabilities associated with 4 constituent susy breaking systems ?

Constituent model

Given the most general extremal BH in a given duality frame E, there exist 4 half-BPS constituents with masses M_i such that the total BH mass equals

$$M_{\text{BH}} = \sum_{i=1}^{4} M_i$$

The identification of the constituents is constructive

1. Identify the $(\mathrm{SL}(2,\mathbb{R}))^3$ transformation bringing the $\overline{\text{D0}}$-D4-D4-D4 system to the frame E
2. Apply such transformation to the 4-constituent charge vectors Γ_i $i = I, II, III, IV$

U-duality guarantees these properties hold anywhere extremal orbit.

[Gimon,Larsen,S]

Constituent model : Probe checks

If there is no binding energy $\Longrightarrow$ add extra quanta with no cost in energy

- if quanta is among our constituent bits (or linear combinations)
- such probes should feel no force

Probe

Using the most general single center extremal black hole, in a given duality frame, as a background in a DBI + WZ effective action describing the propagation of the extra quantum, there exist precise cancellations between gravitational, electromagnetic and scalar interactions giving rise

$$V_{\text{probe}} = V_{\text{DBI}} + V_{\text{WZ}} = 0$$

BPS calculation

The same conclusion does NOT hold for BPS BHs when $B \neq 0$
These are proper bound states, i.e. $\exists$ non-trivial bound energy

Constituent model : Supergravity checks

(particular) Multi-center configurations were found by Gaiotto,Li,Padi

- Location of the centers is unconstrained
- Charges carried by each center match our constituent analysis

These results are consistent with our interpretation
For $B = 0$, the same constituent picture was used to account for the BH entropy, extending the Strominger-Vafa counting argument, using the d.o.f. localised at the D3-brane intersections by Horowitz & Emparan

Comments on BPS vs non-BPS

1. Single center : non-BPS exist everywhere in moduli space; BPS do NOT exist.
2. Multi center : non-BPS exist everywhere in moduli space; BPS only exist for
$$\sum_{i<j} B_i B_j \geq 1$$
3. Distance between BPS centers is fixed
$$R = |\vec{x}_1 - \vec{x}_2| = -\frac{P^0}{\sin\alpha} = \frac{|Q_0 + iP^0 \prod_{i=1}^{3}(1+iB^i)|}{\sum_{i<j} B^i B^j - 1},$$
whereas it is free in the non-BPS case.
4. Constituents are half-BPS in both cases, but they are mutually local in the BPS case, whereas mutually non-local in the non-BPS one
5. Two scalars in the vector multiplet parameterise flat directions of the potential in the non-BPS case, whereas this is never the case in the BPS case.

Almost BPS

1. Goldstein & Katmadas observed that just by $\star_4 \to -\star_4$ in the hyper-Kähler base metric, one could describe extremal non-BPS solutions by similar equations to the one described in the BPS context
2. These were exhaustively studied by Bena, Dall'Agata, Giusto, Ruef, Warner, ...
   - Single center most general solution including angular momentum J
     In the same frame as before, solutions carry the same mass but have intrinsic J
   - Existence of multi-center configurations, involving black rings, BHs ...
     As soon as $J \neq 0$ regularity typically constraints the location of the center
3. Microscopics ? : bound state of three types of 1/4 BPS supertube-bits that mutually break susy

Composite non-BPS

Work by Bossard & Ruef, 1106.5806

Use coset methods and study nilpotent orbits to obtain extremal solutions
Besides recovering previous solutions, new composite non-BPS solutions

Mutually non-local centers

Each center may carry intrinsic angular momentum, but their interaction can generate an extra piece

$$
\begin{aligned}
J &= \sum_i J_i + J_{\text{int}} \\
J^{ij}_{\text{int}} &= P_i^{\Lambda} Q_{j\Lambda} - Q_{i\Lambda} P_j^{\Lambda}
\end{aligned}
$$

Composite non-BPS-II

Charges & regularity

$$2\sqrt{2}M = -\ell_1\ell_2\ell_3 p^0 - m_0 \sum_i \ell_i p^i + \sum_i \frac{1+m_0^2}{\ell_i} q_i \qquad \text{Mass}$$

Asymptotic flatness requires $1 + m_0^2 = h\ell_1\ell_2\ell_3$
Regularity requires $q_0 = 0$ (absence of D0-brane charge) [in their frame]

Fundamental 2-center 'bit'

Minimal two center solution with $J_{\text{int}} \neq 0$

- One center : $\tilde{Q}_1, \tilde{P}^2, \tilde{P}^3$ and a free $\tilde{P}^0$
- Second center : All Q_i, at least 2 P^i and a free P^0

Non-extremal non-BPS

Besides the work already mentioned earlier

- Work by Camps,Emparan, Figueras,Giusto,Saxena describing black rings in Taub-Nut and the interactions of D0-D6 at finite temperature, showing the existence of stable bound states at large enough B field (large enough thermal excitation)
- Partial results on single center and (particular) multi-center configurations
- Very little work in the direction outlined in Jan de Boer's talk
- Interesting work involving probe calculations in non-extremal BHs See talks by Vercnocke, Puhm, Denef

Open questions

Far too many ... besides the ones already mentioned and focusing on the second part of the talk, some more precise directions to explore are

- Quiver gauge theories for extremal BHs
  1. Non-renormalisation properties, stability, 'bubble' equations, ...
  2. IR emergence of CFTs $\Longleftrightarrow$ connection to AdS_2 physics, Kerr/CFT, ...
- Physics of 'extremal' supertubes
- Importance of non-abelian structure
- More general supergravity solutions can allow to study
  1. Wall crossing phenomena in non-BPS set-up
  2. Bubbling picture from a purely AdS_2 and AdS_3 perspective [work by Bobev, Niehoff, Warner]