Flux Compactifications and Black Hole Microstates

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Questions you might have…

Why is there a session on flux compactifications?

Why should I care about black hole physics?

What is the relation of flux comps and BH microstates?

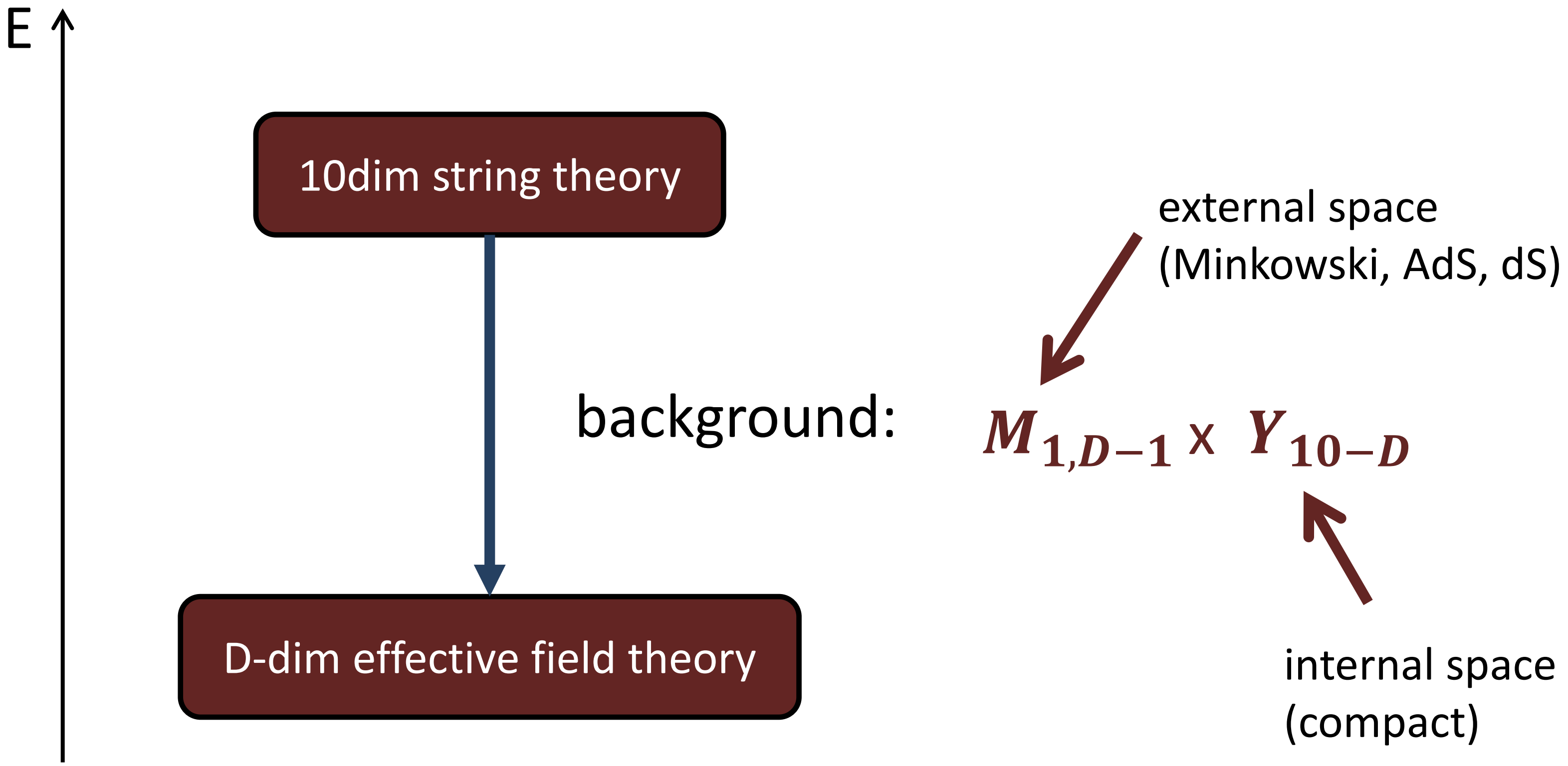
Overview

1. Introduction to flux compactifications

2. Relations to black hole microstates

3. More concrete: An explicit example

String Compactifications



- vacuum = solution to eom with max. symmetry in D dim.

String Compactifications

metric:

$$\mathbf{d}s^2 = e^{A(y)}\mathbf{d}s^2_{\text{Min/AdS/dS}} + g_{mn}\mathbf{d}y^m\mathbf{d}y^n$$

warp factor $A(y)$: can create hierarchies

flux = non-zero background fields that are fixed topologically

We want supersymmetry:

- Control over string corrections
- 1^{st} order differential equations (imply 2^{nd} order eom)
- Add extra ingredient to break supersymmetry (softly)

Calabi-Yau Compactifications

- SUSY condition for type II string backgrounds:

$$\delta_\eta \psi^i_m = \nabla_m \eta^i + \not{H}_m \Gamma^i{}_j \eta^j + \sum_n \not{F}^{(n)}_m \Gamma_{(n)}{}^i{}_j\, \eta^j = 0$$

- Simplest solution: **Calabi-Yau threefold**

$\nabla \eta = 0$ in the absence of fluxes ($H = F^{(n)} = 0$).

$\Leftrightarrow$ Spinor bilinears J and Ω are harmonic

- J and Ω determine metric.
- Harmonic forms give moduli space.

Can we solve this more generally?

Generalized Geometry

Hitchin '04; Gualtieri '03; Witt '04; …

- String theory on T^n has ***SO(n,n)*** symmetry

  "T-duality group"

- Locally, fields transform under **SO(n,n)** in ***any*** background!

Generalized Geometry = SO(n,n) covariant formulation

- In type II and M-theory:

  Hull '07; Waldram, Pacheco '08;
  Graña, Louis, Sim, Waldram '09; …

  $\boldsymbol{E_{n(n)}}$ symmetry **"U-duality group"**

- Simplifications:
  - SUSY equations simple
  - Fields come in $\boldsymbol{E_{n(n)}}$ reps
  - All couplings form $\boldsymbol{E_{n(n)}}$ invariants

See Dan's talk

Supersymmetric Solutions

- More general supersymmetric backgrounds:

  $\nabla\eta^i$ = torsion classes,
  ***H* and $F^{(n)}$ non-zero**

- Supersymmetric solutions have been classified — Graña, Minasian, Petrini, Tomasiello '04, '05;

  SUSY ⇒ Relations between torsion classes, fluxes and warp factor

- An especially useful class of type IIB solutions: — Graña, Polchinski '00; Giddings, Kachru, Polchinski '01
  - Internal geometry is warped Calabi-Yau
  - Warp factor A related to $F^{(5)}$
  - Flux $G_3 = F^{(3)} - \tau H$ is (2,1)

Supersymmetry Breaking

- Add to supersymmetric solution an ingredient that breaks supersymmetry
  - **spontaneously** and
  - at a **low scale**.

  ⇒ Control over corrections induced by susy-breaking

- **By flux:**
  - Eom: G_3 is self-dual
  - SUSY: G_3 is (2,1)

  ⇒ $G_3^{(0,3)}$ not zero ⇔ ~~SUSY~~

- **By Anti-D-branes?**

- …

See Thomas' talk

See Stefano's talk

Effective actions

See
Paul's
talk

- Two motivations:

What is the low-energy theory in D dimensions for a given solution?

Can we find solutions in a simpler D-dim. theory?

- Truncate the theory to a finite set of modes
- Ensure that the solutions for the effective action lift to solutions of the 10/11-dim. action.

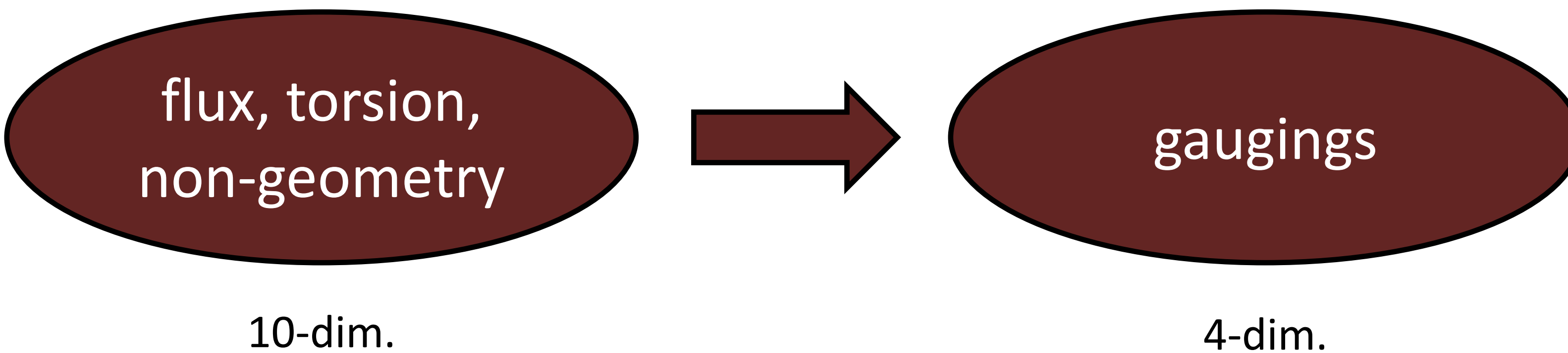
Consistent truncation

- Study properties of this D-dim. action!
- Truncation supersymmetric?

Generalized compactifications

See Diego's talk

- Under mirror symmetry and T-duality, fluxes and torsion map to ***"non-geometric"*** fluxes
- Such non-geometric string backgrounds are not well-understood (maybe **doubled geometry** helps?).
- But their effective action is:



Effective action is a gauged supergravity

What is the relation to black holes?

Black Holes and Black Rings

- Usual form of black hole backgrounds (in M-theory):

non-trivial, non-compact space

$$\underbrace{\boldsymbol{R_t} \times \boldsymbol{M_4}}_{\text{BH/BR}} \times \boldsymbol{N_6}$$

compact space

- There are also M2-branes wrapping two-cycles in $\boldsymbol{N_6}$.
- The $\boldsymbol{G_4}$ flux has usually two legs on $\boldsymbol{N_6}$.
- A further reduction on a circle fiber of $\boldsymbol{M_4}$ gives 4-dim. black hole backgrounds (or the dual IIA background).

Black holes as flux compactifications

- Usual form of black hole backgrounds (in M-theory):

$$\mathrm{d}s^2 = -e^{2A(x,y)}(\mathrm{d}t + k)^2 + e^{-A(x,y)}(g_{ab}\mathrm{d}x^a\mathrm{d}x^b + g_{mn}\mathrm{d}y^m\mathrm{d}y^n)$$

- If rotation k is zero, this looks like a flux compactification on the "internal" space $Y_{10} = M_4 \times N_6$ to one dimension
- On M_4 the fixed asymptotics at infinity replace compactness
- Many tools of flux compactifications do actually **not** require compactness.

Can we use flux compactification techniques to further understand BH backgrounds (and vice versa)?

Generalized Geometry for arbitrary backgrounds

Tomasiello '11

- Start with ten-dim. spinors $\eta^{ai}, i = 1, 2, a = 1, \ldots, n,$ corresponding to left- ($i = 1$) and right-handed ($i = 2$) supercharges (in type II).
- Bispinors $\Phi^{ab} = \eta^{a1} \otimes \bar{\eta}^{b2}$ transform under SO(10,10) as pure spinors. Thus, Φ^{ab} are sums of differential forms!
- SUSY conditions translate into first order differential equations on Φ^{ab} involving fluxes. For $n = 1$:

$$(\mathrm{d} - H\wedge)(e^{-\varphi}\Phi) = -(\widehat{K}\wedge + i_K)F$$

K is lightlike Killing vector of the background

Use power of Generalized Geometry for black holes!

Calibrations

Ferrara, Gibbons, Kallosh '97; Denef '00; Ceresole, Dall'Agata '07; Andrianopoli, D'Auria, Orazi, Trigiante '07; Cardoso, Ceresole, Dall'Agata '07; …

- For static, sperically symmetric (single center) black holes, one can obtain an effectively one-dim. action $\boldsymbol{S}$.
- Often:

$$S = \int \sum_i C_i(\Phi, p, q, A)^2$$

- If all the calibrations C_i vanish, we have a solution (BPS or non-BPS).
- One can find a "real" (or "fake") superpotential $W(\Phi, p, q, A)$ s.t.

$$\mathrm{V} = W^2 + \sum_i (\partial_i W)^2$$

Calibrations and real superpotentials for multicenter solutions?

Calibrations in flux compactifications

See Luca's talk

Lüst, Marchesano, Martucci, Tsimpis '08;
Held, Lüst, Marchesano, Martucci '10

- Calibrations for **general** flux compactifications have been found! Each calibration condition ensures the existence of certain BPS objects in the vacuum.
- Non-supersymmetric solutions are found by violating two calibration conditions such that both terms cancel.
- Generalization of supersymmetry breaking via (0,3) flux

Can one find a real superpotential?

Useful for multicenter solutions?

Anti-D-branes in flux backgrounds

Kachru, Pearson, Verlinde '01;
Bena, Graña, Halmagyi '08; …

- Open question:

Do anti-D-branes break SUSY spontaneously?

Is it a metastable vacuum in the susy theory?

See Stefano's talk

See Thomas' talk

- Can we understand e.g. a Klebanov-Strassler throat with anti-D3-branes using calibrations?
- Can one find a real (or "fake") superpotential?
- There should be a relation to black hole solutions!

Black Holes in Gauged Supergravities

- First BH solutions in gauged supergravity have been constructed recently.

Cacciatori, Klemm ’09; Klemm, Zorzan ‘10; Hristov, Looyestijn, Vandoren ‘10; Dall’Agata, Gnecchi ‘11;

- Can we understand their origin in string theory?

1. Gauged supergravities arise naturally in flux compactifications.
2. Calibration conditions for D-branes in such backgrounds are well-known.

Construct D-brane bound states and microstate geometries in general flux backgrounds?

More concrete:
An explicit example

Bena, HT, Vercnocke ‘11

BPS Microstates of N=8

- Take BPS microstates in 5-dim. N=8 supergravity, coming from M-theory on:

$$R_t \times HK_4 \times T^6$$

- Charges are distributed like:

	0	HK_4	5	6	7	8	9	10
$M2$	X		X	X				
$M2$	X				X	X		
$M2$	X						X	X
$M5$	X	X			X	X	X	X
$M5$	X	X	X	X			X	X
$M5$	X	X	X	X	X	X		

Three-charge solution

BPS Microstates of N=8

- M-theory “compactification” to three dimensions on Calabi-Yau fourfold:

$$\left(R_t \times T^2\right) \times \left(HK_4 \times T^4\right)$$

- Charges are distributed like:

	0	HK_4	5	6	7	8	9	10
~~$M2$~~	~~X~~		~~X~~	~~X~~				
~~$M2$~~	~~X~~				~~X~~	~~X~~		
$M2$	X						X	X
$M5$	X	X			X	X	X	X
$M5$	X	X	X	X			X	X
~~$M5$~~	~~X~~	~~X~~	~~X~~	~~X~~	~~X~~	~~X~~		

Flux “compactification” to three dim.

Dictionary

Flux compactification	Microstate geometry
warp factor	redshift factor
spacetime-filling M2-branes	M2 charges
internal G_4 flux	M5 charges
hyper-Kähler geometry	multicenter Taub-NUT
self-dual two-form	anti-self-dual two-form

Calabi-Yau fourfolds and M-theory

Calabi-Yau fourfold flux compactification: Becker, Becker '96

- Eom: G_4 is self-dual on $HK_4 \times T^4$
- SUSY: G_4 is primitive (2,2)

General solution:

$$G_4 = \sum_i \Theta_+^i \wedge \theta_+^i + \sum_i \Theta_-^i \wedge \theta_-^i$$

include known non-BPS solutions (Goldstein, Katmadas '08)

include known BPS solutions (Gauntlett, Gutowski, Hull, Pakis, Reall '02; Bena, Warner '04; Gutowski, Reall '04)

General G_4 (both BPS and non-BPS) outside of known classifications!

BPS vs. non-BPS

Bena, HT, Vercnocke ‘11

- Solution with primitive (2,2) G_4 is 1/8 BPS in N=8
- It is in general outside of classification of BPS solutions in N=2 supergravity, given in

  Gauntlett, Gutowski, Hull, Pakis, Reall ’02; Bena, Warner ’04; Gutowski, Reall ‘04

- However, it is also a solution of the N=2 truncation
- The reason is that it happens to be non-supersymmetric in **all** N=2 truncations:

1/8 BPS in N=8 → non-BPS in N=2

To summarize

This might be the beginning of an exciting relationship

Flux comps and BHs have similar structure

Generalized Geometry for BH geometries?

Calibrations!

Anti-D-brane backgrounds?

generalized compactifications <-> BHs in gauged SUGRA?

Discover new phenomena